TYPES OF REASONING:
RELATIVIZING THE RATIONAL FORCE OF CONCLUSIONS

K.P.Mohanan
ellkpmoh@nus.edu.sg

Preliminary Draft

This article has three related goals. Firstly, it attempts to present a birds-eye view of what I consider to be the main types of reasoning in both academic knowledge and commonsense knowledge, namely, absolute and probabilistic deductive reasoning, absolute and probabilistic inductive reasoning, abductive reasoning, hypothetico-deductive reasoning, and analogical reasoning. Cutting across the above typology is the distinction between quantitative and qualitative reasoning. Secondly, the article aims to relate the different types of reasoning to the systematization of reasoning in various systems of logic and mathematics. Thirdly, it aims to dispel the widespread tendency to equate "reasoning" with "logic", and "logic" with classical syllogistic logic and its formal descendants.

The overview of the different types of reasoning helps us arrive at an understanding of the notions of "valid reasoning" and "good argument". The reasoning in a good argument must be valid in the sense that the conclusion from the premises must be sanctioned by admissible rules of inference. In addition, however, good arguments must also satisfy the requirements of credibility of premises, confidence of conclusions, and global consistency of arguments.

Table of contents

TYPES OF REASONING:
RELATIVIZING THE RATIONAL FORCE OF CONCLUSIONS*

K.P.Mohanan

1. REASONING

Reasoning may be characterized as the process of *connecting the given information to conclusions, judgments, estimates or inferences*. For instance, from the experience of dark clouds in the sky, we might reason that it is going to rain. From the data on the number of people who smoke and the number of people who have lung cancer, we might reason that smoking is one of the causes of lung cancer. From the axioms of Euclidean geometry, we reason that the sum of angles of a triangle is 180 degrees.

If we use the term **input** as a cover term to refer to experience, data, observations and premises, and the term **output** to refer to inferences, judgments, and conclusions, we may pictorially represent the process of reasoning as follows:

input	dark clouds	data on smokers and cancer patients
	↓ reasoning	↓ reasoning
output	impending rain	smoking causes lung cancer

Figure 1: process of reasoning

We may also think about reasoning as a process that allows us to infer or calculate some information on the basis of some other information. For instance, given the information that Zino is a human being we infer that Zino does not have wings even though the original body of information says nothing about Zino not having wings. Similarly, given the information that Hera's table is a rectangle whose length is four feet and width is three feet, we infer that the area of Hera's table is twelve square feet, even though the input does not contain information about the area of Hera's table.

(**1**) Zino is a human being.
Therefore, we may infer that Zino does not have wings.

(**2**) Hera's table is a rectangle.
The length of Hera's table is four feet.
The width of Hera's table is three feet.
Therefore, we may infer that the area of Hera's table is twelve square feet.

Thus, reasoning may be thought of as the process of making legitimate inferences on the basis of a given body of information, as indicated in figure 2.[1]

* I am indebted to Sunita Abraham for her hawkvision that eliminated an extraordinarily large number of typo's and other goofs in this manuscript. The blame for remaining typo's and goofs obviously goes to her. I have also benefited considerably from Chan Onn's help and corrections on some of the discussions dealing with statistics.

Given information ————————> information
reasoning

Figure 2: reasoning as making inferences

Logic and **mathematics** provide systematized forms of reasoning. In what follows, we explore some of the common modes of reasoning, some of which have been systematized, and others which, as yet, are part of informal practice. The purpose of this exploration is to develop an overall perspective on reasoning, and place the systematized forms of reasoning available in logic and mathematics within this perspective.

3. PREMISES, CONCLUSIONS, AND RULES OF INFERENCE

Reasoning relates one set of propositions to another set. The reasoning in (1), for instance, relates the proposition that Zino is a human being to the proposition that Zino does not have wings.[2]

We use the term **premises** to refer to the input propositions that act as the basis of an inference, and the term **conclusion** to refer to the proposition that is inferred as the output of the reasoning process. In (3), for instance, the proposition that Zino is a human being is a premise, while the proposition that Zino does not have wings is a conclusion.

Quite often, in the actual practice of reasoning, not all the premises that lead to a conclusion are explicitly articulated. The reasoning in (1) and (2) for instance, is incomplete in the sense that one of the crucial premises that legitimize the conclusions is left out since it is considered obvious. The complete statement of the reasoning in these examples is as given in (3) and (4):

(**3**) a. Zino is a human being.
b. Human beings do not have wings.
c. Therefore we may infer that Zino does not have wings.

(**4**) a. Hera's table is a rectangle.
b. The length of Hera's table is four feet.
c. The width of Hera's table is three feet.

---

[1] The orientation of the arrows in figure1 (vertical) and figure 2 (horizontal) is irrelevant.

[2] Logicians and semanticists often distinguish between sentences and propositions. While the term "proposition" refers to the meaning, the term "sentence" refers to the form in which the meaning is expressed. If I say "John pinched Bill." and "Bill was pinched by John.", I am uttering two different sentences, but asserting the same proposition. The sentence "Flying planes can be dangerous" can be interpreted as asserting the proposition that it may be dangerous to fly planes or the proposition that planes that are flying can be dangerous.

This distinction between sentences and propositions parallels the distinction between words and concepts. During Dalton's time, the word "atom" referred to the concept of an indivisible unit of matter, but now the same word refers to a unit composed of a nucleus and electrons. Thus, depending on which concepts are attached to the words in a sentence, it expresses different propositions. Since the English word "God" refers to (at least) two distinct concepts for Christians and Hindus, the sentence "Pat believes in God." asserts two distinct propositions for Hindu's and Christians.

An important ingredient of critical thinking is the ability to unearth and clarify the propositions and concepts expressed by sentences and words.

d. The area of a rectangle is length times width.
e. Therefore we may infer that the area of Hera's table is twelve square feet.

Premise (3b) is **explicit** in (3), but **implicit** in (1). Similarly, premise (4d) is explicit in (4) but implicit in (2). Quite often, as part of the critical evaluation of reasoning in actual practice, we need to unearth the implicit premises that a writer or speaker appeals to in arriving at the conclusions.

When exploring reasoning, the central question that we need to address is that of the legitimacy of conclusions inferred from the given premises. Given that Zino is a human being and that human beings do not have wings, it is legitimate to conclude that Zino does not have wings. In contrast, we would agree that given that Zino is not a human being and that human beings do not have wings, it is not legitimate to conclude that Zino has wings.

Each system of logic sanctions certain types of premise-conclusion pairings as legitimate in terms of a set of rules of inference. For instance, classical deductive logic permits the conclusion in the following inference:

For all x's, if x is a mammal, then x is an animal.
A horse is a mammal.
Therefore it is legitimate to conclude that a horse is an animal.

This conclusion is sanctioned by the rule of inference called Modus Ponens in classical deductive logic, stated as:

Rule of Inference I: Modus Ponens in Classical Logic

Given that:
For all x, if P is true of x then Q is true of x.
and
P is true of x.
it is legitimate to conclude that:
Q is true of x.

Classical deductive logic has an operator that corresponds to the meaning of the English word "all", but it does not have an operator corresponding to "most". Hence, the conclusion in the reasoning given in the example below, though perfectly rational, is **not** sanctioned by any of the rules of inference in *classical* deductive logic.

For most x's, if x is a bird, then x can fly.
A parrot is a bird.
Therefore it is legitimate to conclude that a parrot can fly.

We will get back to the idea of rules of inference sanctioning the premise-conclusion pairings at a later point. For now, what we must remember is that just as a system of mathematics is characterized by the axioms and theories it contains, a system of logic is characterized by the axioms and rules of inference it contains.

3. QUANTITATIVE AND QUALITATIVE REASONING

The example of reasoning in (3) belongs to the domain of logic, while that of (4) belongs to the domain of mathematics. As pointed out above, logic and mathematics are both systematized forms of reasoning. To see the parallel between the two more clearly, let us take another pair of examples.

(**5**) We know that:

Jeona is a horse. All horses are mammals. All mammals are warm blooded creatures.	Given information
Therefore, we may infer that	
Jeona is a warm blooded creature.	Inferred information

(**6**) We know that

Three apples cost two dollars. Four pears cost one dollar. Pat wants to buy six apples and twelve pears from the store.	Given information
Therefore, we may infer that	
Pat must pay seven dollars.	Inferred information

Both (5) and (6) are examples of **deductive reasoning**. The example in (5) illustrates the application of what may be called **classical deductive** logic.[3] The example in (6) illustrates the application of arithmetic.

Classical deductive logic does not make reference to numbers. In contrast, the mathematical reasoning in (4) and (6) crucially makes use of numbers. Because numbers are absent in (3) and (5) but present in (4) and (6), we may say that (3) and (5) are examples of **qualitative** deductive reasoning while (4) and (6) are examples of **quantitative** deductive reasoning. Qualitative reasoning restricts itself to categories, without specifying numbers.

What has been called logic has traditionally been qualitative. A great deal of mathematics involves quantitative reasoning, but mathematics includes qualitative reasoning as well. Set theory and graph theory in mathematics are instances of qualitative reasoning. Even though set theory is taught in mathematics departments and propositional logic is traditionally taught in philosophy departments, they are equivalent formalisms.

In his book *Nature's Numbers: Discovering Order and Pattern in the Universe,* the famous British mathematician Ian Stewart says:

> One of the most striking features of recent mathematics has been its emphasis on general principles and abstract structures — on the qualitative rather than the quantitative. The great physicist Ernest Rutherford once remarked that "qualitative is

[3] Classical deductive logic is a form of reasoning systematized as Aristotelian logic. This type of logic is formalized in modern times as 'propositional calculus'. Its further development called 'predicate calculus'.

just poor quantitative" But that attitude no longer makes sense. Number is just one of the enormous variety of mathematical qualities that can help us understand and describe nature. We will never understand the growth of a tree or the dunes in the desert if we try to reduce all of nature's freedom to restrictive numerical schemes.

(Stewart 1995 , pp 169-170)

To summarize:

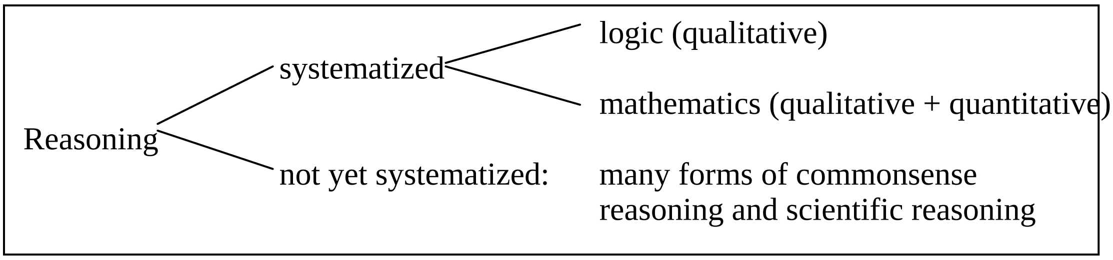


Figure 3: systematized and other forms of reasoning

It is important to bear in mind that "logic" is not the same as "reasoning". Logic systematizes certain forms of reasoning, but not all forms of reasoning come under logic. Only a small area of reasoning is actually systematized as logic. It is also important to distinguish between deductive logic and deductive reasoning. What is called **proof** in mathematics and logic is a special type of deductive reasoning, but deductive reasoning is not restricted to the application of deductive logic. For instance, the reasoning in example 2 is deductive, but it makes use of mathematics, not deductive logic.

4. Deductive and Inductive Reasoning

Most accounts of reasoning distinguish between deductive reasoning and inductive reasoning, illustrated in (7) and (8) respectively.

(**7**) All human beings have ten fingers.
Zino is a human being.
Therefore, we may infer that Zino has ten fingers.

(**8**) All human beings in an observed sample of 10,000 human beings have ten fingers.
Therefore, we may infer that all human beings have ten fingers.

The two examples given above exhibit differences along two parameters, namely, truth preservation and scope expansion.

(**9**) <u>Truth Preservation</u>

A system of reasoning is *truth preserving* if (e.g. (7))
given a set of premises and
a legitimate conclusion ,
the conclusion cannot be false if the premises are true.

A system of reasoning is *non- truth preserving* if (e.g. (8))
given a set of premises and
a legitimate conclusion ,
the conclusion can be false even when the premises are true.

(**10**) <u>Scope Expansion</u>

A system of reasoning is *scope expanding* if (e.g. (8))

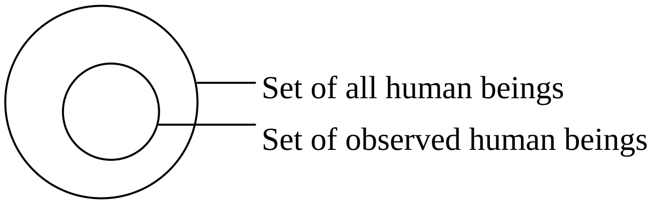
given a set of premises that holds on set A, and
a legitimate conclusion that holds on set B,
set A is a proper subset of set B.

A system of reasoning is *non- scope expanding* if (e.g. (7)

given a set of premises that holds on set A, and
a legitimate conclusion that holds on set B,
either set A and set B are identical,
or set B is a proper subset of set A.

The premises in (7) are that all human beings have ten fingers and that Zino is a human being. The conclusion is that Zino has ten fingers. It is impossible for the premises to be true, and the conclusion to be false. Therefore the reasoning in (7) is truth preserving. The premise in (8) is that all human beings in an observed sample of 10, 00 human beings have ten fingers. The conclusion is that all human beings have ten fingers. In this case, it is possible that the premise is true, and yet the conclusion is false, because there is always a chance that one of the humans that we haven't actually observed has eleven fingers. Therefore the reasoning in (6) is not truth preserving.

Turning to the second parameter, namely, scope expansion, we note that the conclusion in (8) holds on the infinite set of all human beings (set A). In contrast, the premise on which the conclusion is based holds on set B, a proper subset of human beings, that is, the observed sample of 10, 000 humans.



The reasoning in (8) takes information about the proper subset (inner circle) as the given, and arrives at a conclusion about the whole set (the outer circle). Therefore the reasoning in (8) is scope expanding. In contrast, the reasoning in (7) takes information about the whole set as well as one of its proper subsets (Zino), and arrives at a conclusion about the proper subset. Hence, this reasoning is not scope expanding.

Most textbooks on logic use the property of truth preservation to distinguish between deductive and inductive reasoning. By this definition, a system of reasoning is deductive if and only if it is truth preserving. In what follows, however, I will choose an alternative path, and distinguish between them in terms of the property of scope expansion:

(**11**) Inductive reasoning is scope expanding.
Deductive reasoning is scope non-expanding.

The motivation for making this choice will become clear as we go along.[4]

[4] I am restricting the term "induction" to refer to what philosopshers call "enumerative induction", thereby grouping what they call "hypothetical induction", "analogical induction" etc. into separate categories. This is

If we use the terms **population** and **sample** to refer to the whole set and its proper subset respectively, we may say that inductive reasoning sanctions conclusions about a population on the basis of a sample:

Inductive	Deductive		
Sample , Population	Population , Sample	Population x , Population x	Sample x , Sample x

Figure 4: Inductive and deductive reasoning

We stated earlier that different systems of logic are characterized by the rules of inference they contain. All systems of inductive logic share the following rule of inference:

Rule of Inference II: Induction

> If property P is true of a sufficiently large representative random sample of a population,
>
> it is legitimate to conclude that P is true of the whole population.

This rule of inference is absent in deductive logic.

5. VALIDITY:

Given that different systems of logic have different rules of inference, we may define the notion of a **valid** conclusion as follows:

(**12**) Validity

In a system of logic L, a conclusion from a given a set of premises is valid if and only if the conclusion is sanctioned by the rules of inference in L.

In defining validity as (12), we deviate from the standard textbook definition of the term, which is usually given as follows:

(**13**) Standard textbook definition of validity.[5]

A conclusion from a given set of premises is valid if and only if it cannot be false when the premises are true.

Observe that the property crucially appealed to in (13) is that of truth preservation given in (9) above.

The textbook definition of validity applies only to arguments in deductive logic. A non-deductive argument (say, an inductive argument) is generally taken to be neither valid nor invalid. In contrast, the definition in (12) extends the notion of validity to all type of

---

a difference in taxonomy (and terminology). I believe that the taxonomy has an effect on our conceptualization of the issues, but I will not defend my belief here.

[5] Textbooks usually talk about valid arguments, not valid conclusions. We can say that a conclusion from a given set of premises is valid if and only if the conclusion is derived through a valid argument.

reasoning, so that we can talk about inductively valid, deductively valid, analogically valid, and so on.

Let us go back to the inductive inference in (8), repeated below for convenience:

(**8**) All human beings in the observed representative random sample of 10, 000 human beings have ten fingers.
Therefore we may infer that all human beings have ten fingers.

Since the conclusion in (8) is sanctioned by a rule of inference in inductive logic but not by any of the rules of inference in deductive logic, we may say that the conclusion in (8) is valid in inductive logic but not in deductive logic. That is to say, the conclusion in (8) is **inductively valid** but **deductively invalid**.

5. ABSOLUTE AND PROBABILISTIC REASONING

5.1. Probabilistic Deduction and Induction

If we accept the (non-traditional) formulation of the distinction between inductive and deductive reasoning in (11), the examples in (14) come under deductive reasoning, while those in (15) come under inductive reasoning.

(**14**) Deductive reasoning

a. Most human beings have ten fingers.
Zino is a human being.
Therefore we may infer that Zino has ten fingers.

b. Most human beings have ten fingers.
Zino is a human being.
Therefore it is reasonable to conclude that Zino is most likely to have ten fingers.

c. 99.9% of human beings have ten fingers.
Zino is a human being.
Therefore we may infer that the probability of Zino having ten fingers is 0.99

(**15**) Inductive reasoning

a. All human beings in the observed sample of 10, 000 humans have ten fingers.
Therefore we may infer that it is most likely that all human beings have ten fingers.

b. 9,990 human beings in the observed sample of 10, 000 humans have ten fingers.
Therefore we may infer that most human beings have ten fingers.

The reasoning in (14) and (15) is **probabilistic** in that it involves propositions that crucially involve the probability of an event or state of affairs. In contrast, the reasoning in (3) -(8) is non-probabilistic or **absolute**.

To see the relationship between probabilistic and absolute statements, consider the example of a box containing ten balls, one of them black and the remaining nine white. Suppose you are asked to pick a ball from the box with your eyes closed. The probability of your picking a white ball is nine out of ten, while the probability of your picking a black ball is one out of ten (i.e., 0.9 and 0.1 respectively). Suppose you are now asked to pick a ball from a box containing ten balls all of which are white. The probability of your picking a white ball is now ten out of ten and the probability of your picking a black ball is zero out of ten (e.e., 1 and 0 respectively). Statements about the first type of situation are probabilistic, while that of the second type are absolute.

It might be useful to bear in mind that we distinguish between the absolute deductive reasoning in (16) and the probabilistic deductive reasoning in (17):

(**16**) We know that
Fiona is a human being
All human beings have one and only one heart.
Therefore we may infer that
Fiona has one and only one heart.

(**17**) We know that
Fay is a human being.
Human beings generally have ten fingers.
Therefore we may infer that
Fay most likely has ten fingers.

It is equally important to bear in mind that traditional discussions of reasoning reserve the term "deductive reasoning" to refer only to non-probabilistic deduction. We deviate from the textbooks treatments of reasoning in these classifications.

5.2. The Central Rule of Inference in Probabilistic Logics

Both absolute and probabilistic reasoning contain the substance of the following rule of inference:

Rule of Inference III: A different formulation of Classical Modus Ponens

> If property P is true of the entire population,
> it is legitimate to conclude that P is true of the given sample.

However, only probabilistic deduction contains the following rule:

Rule of Inference IV: Probabilistic Modus Ponens[6]

[6] A stronger version of this rule would be:
If property P is true of most of the population,
it is legitimate to conclude that P is true of the given sample.
The reasoning that makes use of the stronger version is fallible however, in the sense that it is not guaranteed to be error free.

If property P is true of most of the population, it is legitimate to conclude that P is most likely true of the given sample.

5.3. Probabilistic Reasoning in Mathematics

As in the case of non-probabilistic reasoning, probabilistic reasoning can also be systematized qualitatively in logic or quantitatively in mathematics. Suppose we are told that there are exactly one thousand CEO's in Albania and that Xu is a CEO in Albania. If we are now told that all the thousand CEO's in Albania are male, we will conclude, using classical deductive logic, that Xu is male:

(**18**) All (the thousand) CEO's in Albania are male.
Xu is a CEO in Albania.
Therefore Xu is male. (=The probability of Xu being male is 1)

Let us change the scenario a little bit. Suppose it turns out that of the thousand CEO's in Albania, only 800 are male. In this case, we cannot be totallyl certain about Xu's gender. The inference about Xu's gender cannot be made in terms of classical deductive logic, but we can appeal to the mathematics of probability:

(**19**) 800 of the 1000 CEO's in Albania are male.
Xu is a CEO in Albania.
Therefore, the probability of Xu being male is 0.8.

Suppose we are not able to count all the male and female CEO's in Albania. However, we do know that most of them are male. Since probability mathematics crucially requires numerical information, we cannot use it to make an inference in this scenario. However, using a probability logic that employs probability quantifiers like "most", we can arrive at the inference: in (20):

(**20**) Most CEO's in Albania are male.
Xu is a CEO in Albania.
Therefore Xu is most likely male. (The probability of Xu being male is high.)

While (19) is an example of quantitative probabilistic reasoning, (20) is an example of the corresponding qualitative probabilistic reasoning.

Inductive reasoning in mathematics is systematized as **statistics**, while probabilistic deductive reasoning is systematized as **probability** theory. While inductive logic and probabilistic logic are systematizations of inductive reasoning and probabilistic reasosning without numbers (qualitative), statistics and probabilisty theory are systematizations of the same with numbers (quantitative).

To see the relationship between qualitative and quantitative reasoning more clearly, consider how we combine probabilistic induction with probabilistic deduction to make an inference on entities which are not part of the original sample. Let us suppose that of 1000 observed crows, 998 are black and the remaining two are white. We are now asked to make an estimate of the colour of a crow outside the original sample. The logician would proceed qualitiatively as in (21).

(**21**) Most crows in the observed representative large random sample of crows are black.
Therefore, it is legitimate to conclude that most crows are black.
Hence, it is legitimate to conclude that the colour of the new crow outside the sample is most likely black

The first step is that of inductive reasoning, while the second step is that of probabilistic reasoning. The mathematician would use actual numbers instead of the quantifier "most". The first step would be that of statistics, and the second step that of probability theory.

In sum, if we adopt the typology of deductive reasoning and inductive reasoning as outlined in (11), we have the following extension of the typology:

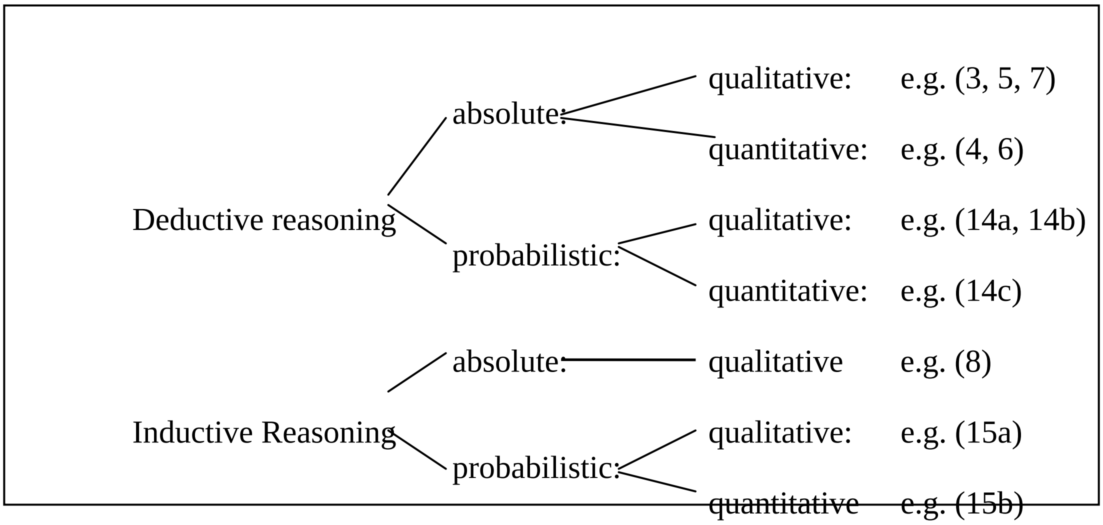


Figure 5: taxonomy of reasoning

These distinctions are best understood by comparing their central rules of inference, repeated below for convenience:

<u>Absolute Deductive</u>
If property P is true of the entire population,
it is legitimate to conclude that P is true of the given sample.

<u>Probabilistic Deductive</u>
If property P is true of most of the population,
it is legitimate to conclude that P is most likely true of the given sample.

<u>Absolute Inductive</u>
If property P is true of a sufficiently large representative random sample of a population,
then it is legitimate to conclude that P is true of the whole population.

<u>Probabilistic Inductive</u>
If property P is true of a sufficiently large representative random sample of a population,

then it is legitimate to conclude that P is most likely true of the whole population.

Just as the term "geometry" is typically associated with Euclidean geometry, the term "logic" is typically associated with the kind of syllogistic logic that Aristotle systematized, namely, qualitative non-probabilistic deductive reasoning. Though perfectly rational, Aristotelian logic does not cover the other types of reasoning in Figure 5.[7]

In the above account of reasoning, we have used a cross-classificatory system. That is to say, one of the parameters of classification is that of scope expansion, which yields the distinction between inductive and deductive reasoning. The other is that of probabilistic and absolute conclusions, which gives us probabilistic and absolute reasoning. When the two parameters are combined, we get four types of reasoning:

	Deductive	Inductive
Absolute	(3), (4)	(8)
Probabilistic	(14a-c)	(15a,b)

Another parameter is that of quantitative avs. qualitative reasoning. When combined with the other two parameters, we should get eight classes, but the figure 5 gives only seven, because inductive quantitative absolute reasoning is missing, because such a system will not be distinct from inductive qualitative absolute reasoning.

We will now go on to explore other parameters on the basis of which we can gain an understanding of the different systems of reasoning.

7. ABDUCTIVE REASONING

Suppose we wake up in the morning and find that the streets are wet. It would be quite reasonable to conclude that it must have rained at night. The reasoning that leads to this conclusion can be stated as follows:

(**22**) If it rained a while ago, the streets would be wet now.
The streets are wet now.
Therefore, in the absence of some other explanation for the wetness of streets, it is reasonable to conclude that it rained a while ago.

Philosopher Charles Sanders Pierce coined the term **abductive reasoning** to refer to the type of reasoning illustrated in (22).[8] This form of reasoning is widely used in every day

[7] Take, for instance, the distinction between crispy and fuzzy logic. Classical deductive logic is said to be "**crispy"** logic because it does not admit degrees of truth. In contrast, **fuzzy** logic, which allows degrees of truth, allows for gradient truth. In addition to being a logic of total certainy, classical deductive reasoning also has the property of being a **two valued** system. That is to say, it incorporates Aristotle's law of excluded middle which says that a proposition is either true or false. A number of **three valued logics** are currently available in which a proposition can be true, false or undecided. Such non-classical deductive logical systems are not part of traditional textbooks on logic. It is therefore important to bear in mind that the kind of logic presented in traditional textbooks is not the only kind of logic available.

[8] I am probably characterizing abduction in a narrower way than Pierce originally intended, but whether or not I am faithful to Pierce does not matter for my purposes. (See C.S. Pierce (1958) *Collected Papers* vol VII, Harvard University Press.)

life and academic disciplines, including the sciences. The kind of reasoning that doctors employ in their diagnosis, for instance, is abductive reasoning:

(**23**) If a person has a heart attack, (s)he will have profuse sweating and pain in the chest, typically extending to the left arm.
Chris has profuse sweating and pain in the chest, typically extending to the left arm.
Therefore, in the absence of some other explanation for the symptoms, it is reasonable to conclude that Chris is having a heart attack.

Abductive reasoning is a way of inferring causes from effects, the inverse of which is classical deductive reasoning. Compare (21) with its classical deductive counterpart in (24):

(**24**) If a person has a heart attack, (s)he will have profuse sweating and pain in the chest, typically extending to the left arm.
Chris is having a heart attack.
Therefore it is reasonable to conclude that Chris has profuse sweating and pain in the chest, typically extending to the left arm.

It is important to recognize that (22) and (23) are not illogical, even though these would be labelled as **fallacies** within the system of classical deductive logic. Thus, if we are using propositional calculus, the reasoning in (25) clearly involves the fallacy of affirming the consequent:

(**25**) P --> Q
Q

Therefore P

If we are using abductive reasoning, however, the reasoning in (23) and (24) is not fallacious, because the conclusions in these examples are sanctioned by abductive reasoning. The rule of inference that sanctions (23) and (24) can be stated as follows:

<u>Rule of Inference V: Abductive Reverse Modus Ponens</u>[9]

If we are given that: P ---> Q Q and the set of premises does not contain P' --> Q it is reasonable to conclude P.

Traditional textbooks on logic do not make this important distinction, and thereby condemn a vast majority of good reasoning as "fallacies".[10] In contrast, we take the

[9] An intuitive way of thinking about this rule is as follows: if we are told that cause C leads to effect E, and we observe E, we are justified in inferring thta C exists, unless we know of some other cause that leads to E.
[10] Note that the rule of abduction is formulated in terms of what lies within our domain of knowledge (that is, the set of premises available to us.). Thus, rule V is **not** the same as rule V':

position that whether or not a given inference is fallacious depends upon which system of reasoning we are using. What is fallacious within one system may not be fallacious within another, and vice versa. If we take the position that a fallacy in a given system of logic is a conclusion not sanctioned by the system, the notion of fallaciousness would then be the opposite of the notion of validity. We can now say that the conclusion in (23) is a deductively fallacious, but abductively valid since it is sanctioned by abductive but not deductive logic. If so, we will have to recognize that an abductively valid conclusion can be deductively fallacious. In other words, the notions of both validity and fallacy are relativized to the system of reasoning in question.

Let us characterize abductive reasoning as a system of reasoning that contains not only Modus Ponens, but also Reverse Modus Ponens. This means that classical deductive reasoning would be a special type of abductive reasoning. Everything that is sanctioned by classical deductive reasoning is also sanctioned by abductive reasoning, but there are inferences sanctioned by abductive reasoning which are not sanctioned by classical deductive reasoning. If so, we may view the two modes of reasoning as **abductive deduction** (i.e., a deductive system that includes reverse modus ponens) and **non-abductive deduction** (i.e., a deductive system that does not include reverse modus ponens).

The rule of abduction given above is formulated in terms of the information available to us. (that is, the set of premises available to us.). Abductive reasoning, therefore, is one of the forms of reasoning that allows us to *make inferences on the basis of limited information.* Note that rule of abduction is **not** the same as rule given below.

(23')If we are given that
P ---> Q
Q
and
there is no P' such that P' --> Q
it is reasonable to conclude that
Q.

(23) contains only two premises, namely, "If P then Q", and "Q". In contrast, (23') contains three premises, namely, "If P then Q", "Q" and "There is no P' such that if "' then Q". The combination of the first and third premise is equivalent to "If Q then P and if Q then P." (23') is therefore expressible in classical deductive logic.

If we are given that
P ---> Q
Q
and
there is no P' such that P' --> Q
it is reasonable to conclude that
Q.

The statement that the input premises do not contain any other other cause-effect relation that leads to Q can be true, and yet the statement that there is no other cause that leads to Q need not be true. The first is a statement about the given premises, the other is a statement about the world. In the terminology of John McCarthy, the declaration that something is not part of the input information is the property of "circumscription".

The statement that the input premises do not contain any other other cause-effect relation that leads to Q can be true, and yet the statement that there is no other cause that leads to Q need not be true. To go back to the example of rain in (22), abductive inference is based on the declaration that the given set of premises does not contain a proposition of the form "If P', then the streets would be wet now." (other than "If it rained a while ago, the streets would be wet now."). We are not introducing the premise that there are no other causes for the streets being wet now. The first is a statement about the given premises, the other is a statement about the world. In the terminology of John McCarthy, the declaration that something is not part of the input information is the property of **circumscription**, which may be metaphorically viewed as drawing a circle around the given set of premises.

Observe that from the point of view of classical logic, abductive reasoning looks dangersously similar to the fallacy of argument ad ignorantiam (argument from ignorance). In their book *Introduction to Logic*, Copi & Cohen characterise this fallacy as "This is the mistake that is committed whenever it is argued that a proposition is true simply on the basis that it has not been proved to be false." (Copi & Cohen (1990: 93)). The arguments in (23) and (24) do crucially appeal to our ignorance of alternative causes, however, they are not based *simply* on ignorance. Hence they do not involve argument ad ignorantium.

8. HYPOTHETICO–DEDUCTIVE REASONING

Consider the following example of reasoning.

(**27**) We observe Brownian motion.
Let us suppose that a liquid consists of jiggling molecules.
From this, Brownian motion would follow.
Therefore, the "jiggling molecules" hypothesis explains Brownian motion.
In the absence of a better theory, therefore, it is legitimate to conclude that a liquid consists of jiggling molecules.

This is an example of what has been called **hypothetico-deductive** reasoning in the literature on the philosophy of science. The structure of (27) can be schematically given as (28):

(**28**) Q	(given information)
Let us suppose P	(hypothesis postulation)
Q logically follows from P.	(deduction)
In the absence of a better candidate, P.	(conclusion)

Hypothetico-deductive reasoning is of particular importance to us because it is used in the argumentation in support of scientific theories. Arguments for theoretical constructs and theoretical laws in the natural sciences follow the pattern given above (arguments for molecules, electromagnetic fields, black holes, curved dimensions, genes...). You will find additional examples of hypothetico-deductive reasoning my article on the evidence for the heliocentric hypothesis. (http://www.nus.sg/Courses/kpmoh/educ/solar.rtf)

Though there is a similarity between abductive reasoning and hypothetico-deductive reasoning, there is also an important difference. The conditional P -- > Q is part of the given premises in abductive reasoning, but not in hypothetico deductive reasoning. Hence, the step "Let us suppose P"in (28) is absent in abductive reasoning:

(**29**) Abductive	Hypothetico-deductive
Q P --> Q. We do not know of P' --> Q such that P' → Q Therefore, we conclude P is correct.	Q Let us suppose P. Q follows from P. We do not know of any P' from which Q would follow. Therefore we conclude that P is correct .

The step "Let us suppose P." is the 'hypothetico' part in hypothetico-deductive reasoning.[11] The following examples illustrate the difference between the two types more clearly.

(**30**) Abductive Reasoning

a. If A and B are "like poles" of two magnets, they would repel each other.
b. A and B repel each other.
c. Therefore, in the absence of an alternative candidate, we are justified in concluding that A and B must be like poles.

(**31**) Hypothetico-deductive Reasoning

a. A and B repel each other.
b. We can explain the above by assuming that if A and B are "like poles" of magnets, they would repel each other.
c. Therefore, in the absence of an alternative explanation for A and B repelling each other, we are justified in concluding that like poles of magnets repel each other.

Abductive reasoning is used in the *application* of a given body of knowledge or theory to a particular situation, as in medical diagnosis or in forensics. It is also found in the application of Newtonian physics to infer the mass of a moving object. In contrast, hypothetico-deductive reasoning is used in the *construction* of the theory itself. In the former, both the observation and and the theory are part of the given information. In the latter, only the observation is given. Thus, when we see an accelerating object, the use of abductive reasoning within the framework of Newtonian physics leads to the inference that there is a force acting on the body. In contrast, if we are using hypothetico-deductive reasoning, we are not tied to Newtonian physics. Rather, we are free to invent any theory that explains why bodies accelerate.

The essential difference between abductive reasoning and hypothetico-deductive reasoning can now be stated as follows. Abductive reasoning allows us to infer causes from effects (abductive) on the basis of a cause-effect relations. In contrast, hypothetico-deductive reasoning allows us to infer the cause-effect relations themselves on the basis of the given effects.

[11] To put it in a slightly technical language, Q is a logical entailment of P in hypothetico-deductive reasoning, while Q is a contingent entailment in abductive reasoning.

10. Analogical Reasoning

The British government has a law protecting animals in experiments, requiring that animals belonging to a privileged class which feels pain should not be operated upon without anesthesia. Now, pain is an internal experience that cannot be directly observed. How do we determine whether a given species of animal is capable of feeling pain or not?

How do each of us know that other human beings are capable of experiencing pain? The strategy is as follows. I know that under certain kinds of external stimuli (e.g. wounds, falls) I feel pain, and when I feel pain, I tend to behave in certain ways (e.g. tears, facial expressions, body postures, noises). When other human beings behave in parallel ways in response to parallel stimuli, I infer that they have the parallel internal experience of pain. This inference is corroborated by what they say about their internal experience. It is therefore justifiable to conclude that human beings are capable of feeling pain.

We extend the same strategy to animals, except that they cannot report their pain to us. If we find that an animal responds in ways parallel to the pain response of human beings to stimuli that parallel pain inducing stimuli for human beings, we conclude that the animal is experiencing pain. The type of reasoning that allows us to make such inferences is **analogical reasoning**.

Analogical reasoning is the process of making inferences on the basis of parallels between two entities or domains. Given below are a few additional examples:

(**32**) Light consists of particles analogous to billiard balls.
When a moving billiard ball collides against a stationary one, the stationary one is set in motion.
Therefore, it is reasonable to conclude that when light particles collide against other particles, the latter are set in motion.

(**33**) The Martians standing in front of me have a face, hands, and legs.
Humans have a face, hands, and legs.
Therefore, Martians are analogous to humans.
Humans are intelligent creatures.
Therefore, it is reasonable to conclude that the Martians standing in front of me are intelligent creatures

(**34**) We cannot fully understand the structure of the heart unless we understand its function. Similarly, we cannot fully understand the structure of language unless we understand its function.

The reasoning in (32) is used in the informal explanation for the photoelectric effect. The analogy of light as a wave is used in the informal explanation for Young's double slit experiment. A great deal of scientific reasoning is analogical.

As far as I know, the inference rules of analogical reasoning have not been formalized or even explicitly articulated. In what follows, I will make a preliminary attempt in this direction, mainly for the purpose of understanding what it involves.

The first step in analogical reasoning involves the intuitive recognition that if two things are parallel, then what is true of one must also be true of the other. This may be stated as follows:

<u>Inference Rule VI:</u> Analogical Truth Transfer (ATT)

If	*w* is analagous to *x*, and
	F is true of *w*,
then	F is true of *x*.

Examples of the use of Analogical Truth Transfer are given in (35)-(38):

(**35**) The genetic code is analogous to a computer programme.
A computer programme is an explicit procedure to be executed in a sequence of steps.
Hence, it is reasonable to conclude that the genetic code is an explicit procedure to be executed in a sequence of steps.

(**36**) The human heart is analogous to a physical pump.
A pump has a piston and a valve.
Therefore, it is reasonable to conclude that the human heart has a piston and a pump.

(**37**) Light is analogous to a billiard ball.
A billiard ball bounces back when it collides with something.
Therefore, it is reasonable to conclude that light bounces back when it collides with something.

(**38**) Human language is analogous to the human heart.
We cannot understand the structure of the human heart without understanding its function.
Therefore it is reasonable to conclude that we cannot understand the structure of the human heart without understanding its function.

Our second rule of analogical reasoning can be stated as follows:

<u>Inference Rule VII: Analogical Extension. (AE)</u>

If	*w* is analogous to *x*,
	y bears the relation R to *w*, and
	z bears the relation R to *x*,
then	*y* is analogous to *z*.

The familiar primary school problem in (39) calls for the use of Analogical Extension.

(**39**) What is x in the following?

2: 4 = 6: x

The problem can be verbally expressed as (40):

(**40**) 2 is analagous to 6.
4 is analogous to x.
2 bears the relation R to 4.
6 bears the relation R to x.
What is x?

There are many solutions to this problem, including those in (41) – (43).

(**41**) 2 is analogous to 6.
4 is analogous to x.
4 is the square of 2.
36 is the square of 6.
Therefore x = 36

(**42**) 2 is analogous to 6.
4 is analogous to x.
4 is the even number that immediately follows 2,
8 is the even number that immediately follows 6.
Therefore x = 8

(**43**) 2 is analogous to 6.
4 is analogous to x.
4 is twice 2.
12 is twice 6.
Therefore x = 12

The third rule of analogical reasoning is that of Analogy Formation:

Inference Rule VIII: Analogy Formation (AF)

If F is true of *w*, and F is true of *x* then *w* is analogous to *x*.

An example of Analogy Formation is given in (44):

(**44**) Those Martians have a face, hands, and legs.
Humans have a face, hands, and legs.
Therefore, Martians are analogous to humans. (by AF)
Humans are intelligent creatures.
Therefore, it is reasonable to conclude that those Martians are intelligent creatures (by ATT)

The combination of the Analogy Formation and Analogical Truth Transer rules is the source of knowledge through empathy. Empathy is an inference about the internal state of an organism through the projection of one's own internal relationships. Take the example of inferring the internal experience of pain in animals mentioned earlier:

(**45**) That dog is a living organism, it responds to stimuli, and it has a face, body, and legs.
I am a living organism, I respond to stimuli, and I have a face, body, and legs.
Therefore the dog and I are analogous. (by AF)
I feel pain when my leg is wounded.
That dog's leg is wounded.
Therefore, it is reasonable to conclude that that dog is feeling pain. (by ATT)

All conclusions about the internal experiences of others (human or non-human) are arrived at through these steps.

The essential characteristic of analogical reasoning is that it allows us to make inferences in one domain on the basis of information from some other domain. Given what we know about human beings, for instance, analogical reasoning allows us to make inferences about dogs. We may refer to this property as that of **domain transfer**.

Many examples of hypothetico-deductive reasoning involve postulating analogical hypotheses and deducing the consequences. Consider, for instance, the following example of hypothetico-deductive reasoning.

(**46**) Light exhibits the phenomenon of interference.
Let us assume that light is analogous to waves.
Waves exhibits the phenomenon of interference.
Therefore light ought to exhibit the phenomenon of interference.
Thus, the assumption that light is analogous to waves explains the interference phenomenon.
In the absence of a better or equally good explanation, therefore, it is reasonable to conclude that light is analagous to waves.

The deduction part of this argument is analogical. If we classify reasoning of this kind as hypothetico-deductive, then it is necessary to take the position that analogical reasoning is a form of deductive reasoning. In other words, we need to talk about **analogical deduction** and **non-analogical deduction**.

10. The Structure of Reasoning

We began our exploration of reasoning with the statement that reasoning is the process of *connecting experience, data, axioms and premises on the one hand to conclusions, judgments, estimates and inferences on the other*, and schematized reasoning in terms of the following diagram:

Given information ————————> inferred information
reasoning

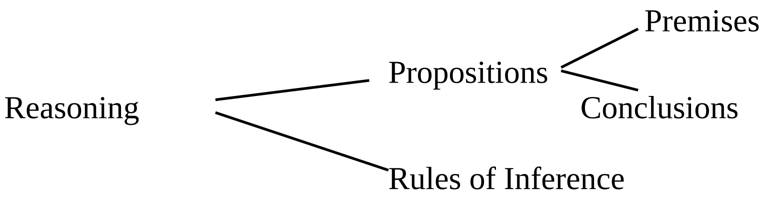
Figure 2: reasoning as inference

Having explored a wide variety of reasoning systems, we are now in a position to take a closer look at the above diagram.

First, given what we said about reasoning as connecting a set of premises to a conclusion, the information in the above diagram can be reformulated as (47):

Propositions (premises)
(**47**) Reasoning ’
Propositions (conclusions)

An important lesson that we have learnt from our examination of reasoning systems is that such premise-conclusion pairings have to be sanctioned by a set of rules of inference that we consider acceptable. Therefore, one of the central ingredients of reasoning is a set of acceptable rules of inference.

(**48**)


Second, we have noted that some of the premises which are necessary for deriving conclusions are quite often left implicit. Thus, the premise that no human beings have wings is not explicitly stated in (1):

(**1**) Zino is a human being.
Therefore we may infer that Zino does not have wings.

Premises which are left implicit in the spoken or written formulation of reasoning are those which are assumed to be shared between the speaker-writer and listener-reader. To understand this aspect of spoken or written expression of reasoning, let us distinguish between **focused premises** and **background premises**. Focus is what the speaker-writer presents to the listener-reader as the basis for arriving at an inference. The rest is background. That is to say, background is what the speaker-reader and listener-reader have already accepted.

Consider the following scenarios:

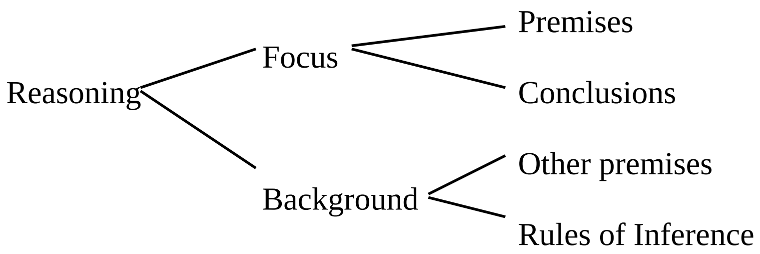
(**49**) speaker: I believe that Zino does not have wings.
listener: Why do you believe that Zino does not have wings?
speaker: Because no human beings have wings.

(**50**) speaker: I believe that Zino does not have wings.
listener: Why do you believe that Zino does not have wings?
speaker: Because Zino is a human being.

In scenario (49), the speaker offers the statement “No human beings have wings” as the basis of the conclusion that Zino does not have wings. Hence, the proposition that no human beings have wings is the focus, while the proposition that Zino is a human being is the background. In (50), the situation is reversed: the proposition that no human beings have wings is the focus, while the proposition that Zino is a human being is the focus.

Since the background is something that the speaker-writer and listener-reader have already accepted, we may include not only the shared premises, but also the shared rules of inference as part of the background for a piece of reasoning. See (51):

(**51**) Structure of reasoning



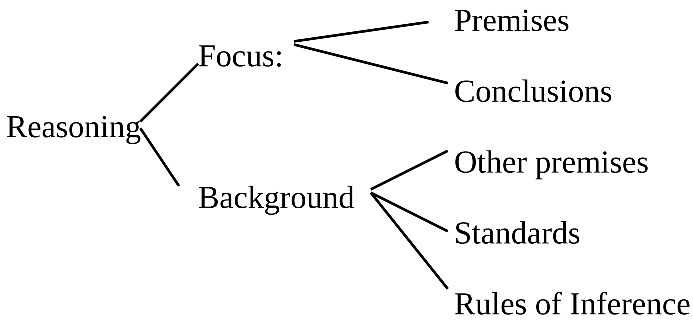
In addition to rules of inference, many instances of reasoning also involve what may be called epistemological **standards** of truthhood within a given community. In the community of natural scientists, for instance, the standards of truth for theories involve the following criteria:

A theory should make correct predictions
make no incorrect predictions
be logically internally consistent
be consistent with other theories
be as general as possible
be as simple as possible

There is no requirement that a theory should be consistent with scriptures or with some authority. In contrast, the reasoning in many communities of theologicians crucially involves the condition of consistency with scriptures. Most humanities communities impose the condition of internal consistency, but not the condition of correct predictions.

Given what we have said above, we may expand the picture in (51) as follows:

(**52**) Structure of Reasoning



Critical evaluation of reasoning crucially involves the scrutiny of not only the focus premises and the conclusion, but also the background premises, standards, and rules of inference. Since the background is generally left unstated, the reader-listener needs to unearth it and crystallize it before examining its reliability.

In the framework of philosopher Stephen Toulmin, a piece of reasoning consists of a set of grounds, warrants, and claims. Toulmin's **grounds** corresponds to our focus premises.[12] His **claims** and **warrants** correspond to our conclusions and background premises respectively.

Notice that focus premises, background premises, and conclusions can be judged to be true or false. In contrast, standards and rules of inference are neither true nor false, but simply good or bad (acceptable or unacceptable). Whether we call it background or warrant, what is important is the need to unearth the implicit aspect of the background/warrant, so that we can critically evaluate the credibility of background premises, and the acceptability of standards and rules of inference.

11. COPING WITH UNCERTAIN KNOWLEDGE

Fuzziness

Suppose I tell you that Pat Jenkins is five feet tall. You measure Pat's height and discover that Pat is actually five feet and one inch. Is my statement that Pat Jenkins is five feet tall false?

The answer to the question depends upon how you interpret the sentence "Pat Jenkins is five feet tall." If you interpret the sentence as saying that Pat Jenkins is exactly five feet tall, not a fraction of an inch more or less, the sentence is clearly asserting a false proposition. On the other hand, if you interpret the sentence as saying that Pat Jenkins is five feet tall, give or take an inch or two, the sentence asserts a true proposition. It will be false only if Pat Jenkins is actually, say, five foot six.

If I tell you that Pat is tall, the situation gets worse. If Pat is four feet tall, surely my statement is false, and if she is six feet tall, my statement is true. In between four feet and six feet is a gray area that the word *tall* refers to.

---

[12] S. Toulmin, R. Rieke, & A. Janik (1979) *An Introduction to Reasoning*, Macmillan Publishing Co.

The issue illustrated by the above example is that of **Fuzziness**. In both everyday knowledge and academic knowledge, we need to make assertions about the world that lack total exactness. Even in the so called “exact sciences” like physics and astronomy, the claims we make about the world need to recognize a certain amount of gray area. This margin of error is explicitly acknowledged in quantitative reasoning in the statements about standard deviation, margin of error, confidence interval, and so on. In qualitative reasoning, we acknowledge it using words like “almost all”, “most”, “generally”, “most likely”, and so on.

Given the information that all CEO’s in Algex women, and that Pat Jenkins is a man, we can infer that Pat Jenkins is not a CEO. This inference can be captured in traditional “crispy” deductive logic. Given the informatin that all CEO’s in Algex are tall, and that Pat Jenkins is five feet tall, we can infer that Pat is not a CEO. This inference cannot be captured in traditional deductive logic. What is called fuzzy logic was designed to capture the legitimacy of such inferences.

Fuzziness, or the absence of total exactness, is part and parcel of human knowledge. We may view fuzziness as a form of **uncertainty**. Another form of uncertainty is that of fallibility, to which we turn in the following section.

Fallibility

Consider the following statements:

(**53**) a. The sun always appears in the east in the morning.
b. The sun always appears in the west in the morning.

(**54**) a. Human beings have one head.
b. Human beings have two heads.

We reject both (53b) and (54b) as clearly false. What about (53a) and (54a)? Are we justified in believing that they are true?

The answer depends on what we mean by the term “justification”. If justifying a claim requires proving that the claim is true, (53a) and (54a) by definition are not justified. We can prove that within the framework of Euclidean geometry, the sum of angles of a triangle is 180 degrees, but we cannot prove (53a) or (54a). On the other hand, if justifying a claim means providing reliable evidence to support the claim, then both (53a) and (54a) are justified. On the basis of observing a large number of instances of the sun rising in the east and never in the west, we consider ourselves justified in concluding that the sun always appears in the east in the morning. Similarly, on the basis of observing a large number of human beings with one head, and having never observed a human being with more than one head, we consider ourselves justified in concluding that all human beings one and only one head. Such conclusions are made on the basis of inductive reasoning, which we characterized as arriving at conclusions about a population on the basis of a sample.

Are we totally certain that the sun will not appear in the west in the morning some day? Are we totally certain that we will not find a two-headed human being some day? The answer is no. We are reasonably certain, but not totally certain. This means that the beliefs that the sun will always appear in the east in the morning, and that there are no two-headed humans are **fallible**, that is, subject to the possibility of turning out to be false. In other words, no belief that we consider to be knowledge of the world can be proved to be true beyond the shadow of doubt.

Dealing with Fuzziness and Fallibility

Two of the important characteristics of inductive reasoning can be stated as follows:

A. Inductive reasoning cannot yield total certainty.
B. Certainty of conclusions in inductive reasoning increases with increase in the size of the sample.

Consider the familiar example of inductive reasoning in (55):

(**55**) Bird 1 is a crow, and it is black. Bird 2 is a crow, and it is black. Bird 3 is a crow, and it is black. ... Bird 1000 is a crow, and it is black.
Therefore, all crows are black.

That the one thousand crows observed so far are black does not provide total guarantee that the next crow to be observed is going to be black, which is why we say that inductive reasoning is not truth preserving. As a result, it cannot yield total certainty of conclusions (characteristic (A)).

We can signal the absence of certainty in the we way we formulate our conclusion:

(**56**) Bird 1 is a crow, and it is black. Bird 2 is a crow, and it is black. Bird 3 is a crow, and it is black. ... Bird 1000 is a crow, and it is black.
Therefore, until there is evidence to the contrary, it is reasonable to conclude that all crows are black.

To turn to characteristic (B), let us note that although we are not absolutely certain that there are no white crows, we still are justified in believing that there are no white crows. Let us suppose that the sample size is ten thousand instead of a thousand. There is greater certainty that all crows are black. If we observe one million crows, our certainty is still higher. Yet, however large the sample we observe, our certainty is never absolute, unless the sample is the same as the population.

Statisticians use the term **confidence** to refer to the strength of an inference or belief. To understand the concept of confidences, let us begin by using the symbol p to refer to the proportion of black crows in a sample. If 800 out of 1000 crows in a sample are black and the rest are non-black, the proportion p of black crows in the sample is 800/1000, that is, 80%. We use the number 0.8 to indiate that the proportion is 80%. If 950 out of 1000 crows are black, the proportion p is 0.95. If 1000 out of 1000 crows are black, p = 1.

Let us use the symbol p_0 to refer to the proportion of black crows in whole population of crows. The question that we are interested in is: given that the proportion p of black crfows in the sample is 1, what is the likelihood of the proportion p_0 of black crows in the entire population also being 1? (i.e., that is the likelyhood of all crows being black?); what is the likely hood of p_0 being, say 0.9 or 0.99?

If we use the symbol C to refer to the confidence in our estimate, and the sumbol n to refer to the size of the sample, the answer to the above question is given by the following formula:

$C = 1 - p_0^{n+1}$

Suppose $p_0 = 0.99$. That is, we are interested in finding out the likelyhood of 99% of the population of crows being black. Then our confidence C in the estimate that 99% of the population of crows is black is as follows:

$$C = 1 - p_0^{n+1} = 1 - (0.99)^{1000} = 0.999957.$$

Suppose we want to be more assertive and set $p_0 = 0.999$. That is, we are interested in finding out the likelyhood of 99.9% of the population of crows being black. If so,

$$C = 1 - p_0^{n+1} = 1 - (0.999)^{1000} = 0.0.6327.$$

This is much lower than the case above since we are making a more stringent statement. Let us now take an extreme case. Given that 1000 out of 1000 crows in the given sample are black, what is our confidence in the estimate that all crows are black, that is the proportion of black crows in the whole population is 100%? The answer is:

$$C = 1 - p_0^{n+1} = 1 - (1)^{1000} = 0$$

We have zero confidence in the estimate that **all** crows are exceptionlessly black.

To summarise, given that 1000 out of 1000 crows are black, we have 0.999957 confidence that 99% of the population of crows is black, 0.0.6327 confidence that 99.9% of the population is black, and zero confidence that 100% of the population is black. Thus, given that all the crows in a given sample are black, the mathematical formula does not permit us to infer that all crows in the population are black. If so, given a crow outside the sample, we cannot be totally certain that it is black. Thus, we derive the result in (A), namely, that *inductive reasoning cannot yield 100 percent confidence, that is total certainty*.[13]

To turn to property (B), we note that when the sample size n increases, n+1 increases. Given that p_0 is less than 1, increase in n+1, decreases the value of p_0^{n+1} , and hence increases the value of C. For example:

n \ p_0	0.99	0.999	0.9999
1000	0.999 957	0.632 672	0.095 258
1500	0.999 999 72	0.777 260	0.139 385
2000	0.999 999998	0.864 935	0.181 359
10000	almost 1	0.999 955	0.632176

Using the notion of confidence, the statistician's version that forms the quantitative counterpart of (56) can be stated as (57):

[13] This result is called Hume's "problem" in philosophy. Strictly speaking, this should be called Hume's result, rather than Hume's problem, since all that it says is that there is always an element of uncertainty in inductive reasoning. It becomes a problem only if we demand absolute certainty of knowledge.

(**57**) Every crow in the random sample of 1000 crows is black.
Therefore, it is reasonable to conclude that the confidence in proposition that the proportion of black crows in the whole population of crows is p_0 is C.

While (56) is an example of inductive logic in which conclusions are fallible (signalled by the phrase “until there is evidence to the contrary” in the conclusion), (57) recognizes the uncertainty by specifying the degree of confidence. While (56) asserts a radical conclusion at the risk of being wrong, (57) asserts a modest conclusion that is unlikely to be totally false provided the sample size is large enough. We will therefore refer to (56) as **radical induction** and (57) as **conservative induction**.

Typically, inductive inferences in the paradigm of theoretical research tend to be of the radical type, where the researcher takes the risk of the conclusion turning out to be wrong. In contrast, inductive inferences in the paradigm of experimental research tend to be of the conservative type, where he researcher plays it safe.

Let us suppose that 998 crows in the observed sample of 1000 are black, and the remaining two are white. The conclusion in qualitative inductive reasoning would be (56), while that of the quantitative reasoning of a statistician would be (57).

We may think of fuzziness and fallibility as different aspects of **uncertainty**. We stated earlier that while the conclusions in classical deductive reasoning are absolutely certain, those in probabilistic deductive reasoning are less than absolutely certain. Probabilistic reasoning lacks exactness, while inductive reasoning has a choice between inexactness (as in (57) and fallibility (as in (56)). We may therefore say that probabilistic reasoning and inductive reasoning are both forms of uncertain reasoning, in the sense that they do not yield conclusions with total certainty. In what follows, we explore a few other aspects of uncertain reasoning.

Non-monotonicity

Suppose you are told that Pat Jenkins is the brightest student in his school. What are the chances that he will pass the examinations with a good grade? You will conclude that the chances are high. Suppose you are now told that Pat is also extremely lazy, and hardly every studies or even pays attention to the teacher in the class. What are the chances that he will pass the examinations with a good grade? You are likely to reverse your conclusion and say that the chances are very low.

Suppose you are told that Pat Jenkins is a secretary in Algex Corporation. Was Pat at home on 11 am last Monday? It would be reasonable to conclude that he was not. You are now given the additional information that Pat Jenkins was very sick last Monday. The conclusion that Pat was not at home at 11 am is no longer reasonable. Additional information, in other words, can change the reasonableness of conclusions.

These examples illustrate the need to revise our earlier conclusions when new information becomes available. A system of reasoning that allows such reversals is said to be **non-monotonic**.

A non-monotonic system has the property that a valid conclusion (in the sense defined in (12)) based on a given body of information can cease to become valid when additional information is provided. A monotonic system of logic contains the following rule of inference:

(58) Monotonicity:

$$(P \neq Q) \div ((P \& R)) \neq Q).$$

The single arrow in (58) expresses ordinary entailment, while the double arrow expresses the logical entailment of the rule of inference. What (58) says can be stated as:

If the proposition "if P then Q" is true,
the proposition "if (P & R) then Q" is also true.

In a monotonic system, therefore, new premises cannot retract the validity of a prior conclusion made on the basis of a given set of premises. In a non-monotonic system, this result does not hold. As we have seen, abductive reasoning, hypothetico-deductive reasoning and analogical reasoning are non-monotonic.

Non-monotonic reasoning may be viewed as *reasoning with limited information, with fallible inferences that change under new information*. This characteristic is a fundamental ingredient of both common sense knowledge and academic knowledge, and particularly so of scientific knowledge. Conclusions in the domain of theoretical science are accomapanied by signalling phrases that refer to available information:

<u>In the absence of counterevidence</u>, it is reasonable to conclude that X is correct.
<u>In the absence of better or equally good alternatives</u>, it is reasonable to conclude that X is correct.

The underlined phrases devices of circumscription mentioned in section 9.

Non-uniqueness

In classical logic, the conclusions sanctioned by the rules of inference from a given set of axioms cannot be logically contradictory. A reasoning system that has the property of **non-uniqueness** of inference permits alternative (contradictory) conclusions for the same body of evidence each of which is legitimate/valid by itself.

Analogical reasoning is non-unique. Suppose you are given the following number series:

<u>Series A</u>: 3 4 5 **6** 5 4 3 2   <u>Series B</u>: 4 5 6 7 8 9 8 7 6

Take number 6 in series A. What is the corresponding number in series B? Most of you would probably say 9, since 6 is the highest number in A, and 9 is the highest number in B. However, the answer 7 is equally correct, since 6 is the fourth number in A and 7 is the fourth number in B. Similarly, one can say that since 6 is the second even number in A, the corresponding number in B is 6. All of these conclusions are equally valid, since they are sanctioned by the rules of analogical reasoning (see also the examples in (41)-(43).

It is well known that any given body of data can be explained by more than one competing scientific theory. In other words, hypothetico-deductive reasoning validates arguments for competing theories on the basis of the same data. Therefore, this mode of reasoning also has the property of non-uniqueness.

Uncertainty of human knowledge

What we call "knowledge of the world" is essentially a set of interpretations of the sensory measurements registered on our biological sensors (retina, tympanum, skin, muscles, etc.) At the lowest level, these interpretations are sensory perceptions. The human brain, for instance, interprets information from retinal sensations as objects in the external world. The sensory interpretations are further interpreted as generalizations and theories about the world. The knowledge constructed in this fashion is inherently uncertain: it is both inexact, and fallible. Furthermore, the reasoning that we employ in arriving at conclusions about the world is non-monotonic and non-unique. Hence, it is important that the systematized forms of reasoning that captures the rationality of the human knowledge of the world be able to express fuzziness, fallibility, non-monotonicity, and non-uniqueness.

The Greek tradition and the subsequent enlightenment tradition in Europe were in search of totally certain infallible knowledge. As a result they over-emphasized the role of absolute crispy monotonic deductive reasoning. This state of affairs continued till the end of nineteenth century. The explicit acknowledgement of the uncertainty of human knowledge, and the development of various systems of uncertain reasoning in the construction of uncertain knowledge, are the hallmarks of twentieth century science.

12. From Valid Reasoning to Good Arguments

Reasoning and Argumentation

From the notion of reasoning, we now move on to the notion of argument. An argument is a piece of reasoning with the purpose of convincing someone of the correctness of a conclusion is credible. A conclusion offered in this manner is a claim. Thus, the pairing of premises and conclusions in a piece of reasoning becomes an argument when the conclusion becomes a claim that we wish to be accepted.

An argument crucially involves a **presenter** (the one who presents the claim) and an **evaluator** (one who critically evaluates the claim to decide whether the claim should be accepted or not. An evaluator may be an individual (as in the conversation between two people), an audience (as in an oral presentation of a paper) or a community (as in the written presentation of a paper). We may think of presenters as writers and speakers, and evaluators as readers and listeners.

The question we wish to address in this section is: what is a good argument? Before we attempt an answer, we need to review our notion of valid reasoning first.

Validity Revisited

In section 5, we defined the notion of *valid conclusions* relative to a given system of reasoning:

(**12**) In a system of logic L, a conclusion from a given a set of premises is valid if and only if the conclusion is sanctioned by the rules of inference in L.

Given our conception of reasoning as inferring a conclusion from a set of premises, we may characterize *valid reasoning* as follows:

(**59**) In a system of logic L, an instance of reasoning is valid if and only if the conclusion from the given premises is sanctioned by the rules of inference in L.

We may now take a step further, and define validity relative to an evaluator, the individual or group engaged in the activity of critically evaluating a given knowledge claim. What does an evaluator consider valid? We may give the following answer:

(**60**) A conclusion from a given given set of premises is valid if and only if it is sanctioned by a set of rules of inference acceptable to the evaluator.

(**61**) An instance of reasoning is valid if and only if the conclusion is sanctioned from the premises by a set of rules of inference acceptable to the evaluator

When defined as (61) and (62), the notion of validity is equivalent to the notion of the "rationality" or "reasonableness" of a conclusion or sample of reasoning.

Goodness of Arguments

What is a **good** argument? Let us begin by saying that *a good argument is one that has the effect of making an evaluator accept a conclusion as correct.* The question is, what are the properties of arguments that trigger the acceptance of a conclusion on the part of an evaluator?

Clearly, the reasoning in a good argument must be valid argument. Therefore validity of reasoning is a necessary condition for good arguments. But are all arguments with valid reasoning good arguments?

In the domains of mathematics and logic, the only requirement on good arguments is that they be valid. In domains that deal with empirical knowledge, however, a good argument must satisfy additional criteria. Classical logic, for instance, requires empirical arguments to be sound:

(**62**) <u>Classical notion of soundness</u>

An argument is sound if and only if
- (a) its reasoning is valid, and
- (b) its premises are true.

Now, other than the strategies of knowledge critiquing approved by an individual or community, we have no independent way of telling whether a given proposition about the world is true or not. Hence the classical condition on truth is not a viable one. A way out of this difficulty is to relax the condition on true premises, and impose the weaker demand the premises be credible to the evaluator

(**63**) <u>Revised notion of soundness</u>

An argument is sound if and only if
- (a) its reasoning is valid, and
- (b) its premises are accepted as true by the evaluator.

It is easy to check whether or not a proposition satisfies (63b). In contrast, I do not see how we can check whether a proposition satisfies (62b) or not. If we define soundness as (62), there will not be a single argument in science or any other domain for that matter which can be considered sound.[14]

Given (63), we can say that good *emprical* arguments must be sound. This statement points to two of the fundamental differences between mathematical sciences and empirical sciences. First, the notion of validity required in mathematics is that of deductive validity: the only form of argument admissible in mathematics is that of proof.[15] In contrast, empirical arguments permit inductive, abductive, hypothetico-deductive and analogical validity. The second difference is that mathematical arguments do not require condition (63b). The premises in empirical sciences are propositions about observations which are claimed to be true about the world. In contrast, the premises in mathematics are axioms which make no claims about the world. Hence, mathematical arguments are not required to be sound. Thus, the notion of good arguments in empirical sciences is more stringent than that in mathematics in that the former requires both (63a) and (63b) while the latter requires only (63a), but the notion of validity itself (63a), is more stringent in mathematics than in empirical sciences in that mathematics permits only deductive validity.

Turning our attention to empirical argumentation, we may summarise the preceding discussion as follows. For an empirical argument to be considered good:

A. its reasoning must be valid (in the sense defined in (61)).

B. the premises should be credible to the evaluator; that is the critical evaluator must judge the premises to be true.

In the previous section, we introduced the notion of strength of confidence in quantitative inductive reasoning. The counterpart of statistical confidence in abductive reasoning and hypothetico deductive reasoning is the convergence of evidence from independent sources: if we find evidence for X from a large number of unrelated sources, our confidence in the belief that X is true gets strengthened.[16] If we take the position that conclusions with poor confidence level are unacceptable, confidence becomes an important ingredient in any system of reasoning that incorporates some form of uncertainty. Even though conclusions which are not totally certain are nevertheless legitimate, we need to say that the degree of uncertainty must not exceed a certain limit. Hence, for an empirical argument to be considered good:

C. the conclusion must have a sufficiently high degree of confidence under the canons of confidence acceptable to the evaluator.

Finally, all forms of empirical knowledge call for a global condition on logical consistency: the totality of knowledge propositions for any given individual or community must be logically consistent with one another. If one set of acceptable premises sanction conclusion

[14] According to Copi and Cohen, for instance, "When an argument is valid and *all* of its premises are true, we call it "*sound*". The conclusion of a sound argument obviously must be true." (I. Copi & C. Cohen (1990:52) *Introduction to Logic*. Macmillan. Given their notions of validity and soundness, it follows that the arguments for theoretical laws and constructs in science are not sound.

[15] See the distinction between arguments as proofs and other types of arguments in my article "Modes of Justifying Knowledge Claims". (http://www.nus.edu/Courses/kpmoh/critical/justific.rtf).

[16] See the discussion of convergence of evidence as 'objectivity' in my article "Modes of Justifying Knowledge Claims. (http://www.nus.edu/Courses/kpmoh/critical/justific.rtf).

X, and another set of acceptable premises sanction conclusion not X, then the argument for both X and not X are inconclusive. This situation calls for an additional requirement on goodness in argumentation. For an empirical argument to be considered good:

D. the conclusion must be consistent with (that is, should not logically contradict) the other conclusions judged to be true by the evaluator.

In sum, we may say that for an empirical argument to be considered good, it must satisfy the requirements of (a) validity of reasoning, (b) credibility of premises, (c) confidence of conclusions, and (d) consistency of conclusions with other conclusions.

It is possible that the list given above is not exhaustive, and that other conditions must be added to the list to characterize the notion "good argument" across academic communities. If so, what I have given in A-E must be viewed as a starting point for an enquiry.

Validity and Standards of Comparison

In the previous section, we pointed out that non-monotonic reasoning is that lends itself to reasoning under liminited information. It allows us to say, given such and such information, it is reasonable to conclude such and such, however, the conclusion may change when new information becomes available. The rules of inference in both hypothetico deductive reasoning contain the cautionary phrases underlined below:

In the absence of
(i) counterevidence, and
(ii) better or equally good alternatives,
it is reasonable to conclude that X is correct.

Given (i), it follows that when counterevidence becomes available, the conclusion that we took to be valid under old information is no longer valid. Similarly, given (ii), it follows that when an equally good or better alternative becomes available, the prior valid conclusion becomes invalid. Thus, a conclusion that has been established on the basis of abductive and hypothetico deductive reasoning can be refuted either by demonstrating the existence of counterevidence ((i)), or the existence of better alternatives.

In order to apply specification (ii) it is necessary to have a set of critieria or standards by which we decide whether two explanations are equally good, or one is better than the other. As mentioned earlier, two such standards in the comparison of theories in natural sciences are simplicity and generality:

Generality: Everything else being the same, explanations which are more general are better than less general explanations.

Simplicity: Everything else being the same, simpler explanations are better than less simple explanations.

In sum, in those forms of reasoning where the conclusions are non-unique, validity of reasoning presupposes a set of **standards** on the basis of which we choose between **competing alternatives**.

Implicit and ExplicitValidity

Finally, let us turn to the issue of explicitness in argumentation. We have already seen that in real life, arguments to do not often spell out all of its premises in an explicit manner. When the premises are too obvious to be mentioned, the presenter often leaves them out, assuming that the missing premises can be supplied by the evaluator. The question is, do the missing premises make an argument defective?

Let us go back to our first example of reasoning, repeated below for convenience.

(**1**) Zino is a human being.
Therefore, we may infer that Zino does not have wings.

This is not an example of defective reasoning. In contrast, we have no hesitation in acknowledging the reasoning in (**64**) to be defective:

(**64**) Zino is a human being.
Therefore, we may infer that Nina does not have blue eyes.

The reasoning in (1), even though technically not valid yet, is potentially valid in the sense that the evaluator can easily supply the missing premise and then accept the reasoning in (3) as valid.

(**3**) a. Zino is a human being.
b. Human beings do not have wings.
c. Therefore we may infer that Zino does not have wings.

The parallel treatment is impossible for (64). We may therefore treat both (1) and (3) as valid, the distinction between them being that while (1) is **implicitly valid**, while that in (3) is **explicitly valid**. In contrast, the reasoning in (64) is **invalid**.

Different evaluators have different levels of tolerance for what can be taken for granted and hence be left implicit. A phonologist writing for fellow phonologists, for instance, would take a large number of theoretical hypotheses as widely accepted and hence would not argue for them or even explicitly articulate them in an argument for a controversial claim. In contrast, a phonologist writing for non-phonologist linguists will have fewer implicit premises. A phonologist writing for non-linguists will have still fewer implicit premises.

Non-explicitness of premises by itself is not a defect.[17] What makes implicitness defective is the degree of difficulty for the evaluator in unearthing the presenters implicit premises in a clear and unambiguous manner.

What I have said above about implicit premises also applies to implicit rules of inference and implicit standards (See the section on the structure of reasoning). Implicitness of inference rules and standards will not be considered defective as long as the evaluator can supply the missing pieces. If not, the reasoning in the argument will be judged to be invalid.

Two practical consequences from what we have said above. First, a good writer (presenter) is one who can correctly estimate of the premises, rules of inference, and standards that are considered admissible by the evaluator. Conversely, a good reader (evaluator) is one who can unearth and articulate the hidden premises, rules of inference, and standards of the

[17] Over-explicitness of obvious premises may in fact lead to what may be perceived as bad or tedious writing.

writer. These are requirements on the communication of reasoning in a text, not on reasoning per se.

13. CONCLUDING REMARKS

To summarize, our exploration of the different types of reasoning unearthed the following parameters of distinguishing between different types reasoning:

The use of numbers:	Qualitative vs. Quantiative
Scope expansion:	Inductive vs. Deductive
Truth preservation:	Classical deduction vs. everything else.[18]
Expression of probability:	Probabilistic vs. Absolute
Inferring causes:	Abductive deduction vs. non-abductive deduction
Inferring cause-effect relations:	Hypothetico-deductive vs. abductive
Domain transfer:	Analogical vs. non-analogical

Truth preservation and probability are tied with the property of uncertainty. Fallibility, fuzziness, non-monotonicity and non-uniqueness are the different types of uncertainty.

What is important in the above discussion is not the way we have classified and labelled different types of reasoning, but the rules of inference that characterize different modes of reasoning, and an appreciation of the kinds of conclusions that are legitimized by each. For instance, it may be the case that some books use the term "inductive" reasoning to refer to what we have called probabilistic deduction, analogical deduction, abduction, and hypothetico-deduction. This means that their classification and labelling differ from ours. I prefer the classificatory system that I have outlined in the previous sections because it matches the intuitive meanings associated with "induction" and "deduction", but the difference is nevertheless a superficial one.[19]

I have suggested that the notion of validity should be defined relative to reasoning systems, such that we can say that certain premise-conclusion pairings are valid under reasoning system X, but invalid under reasoning system Y. I have also suggested a similar treatment of logical fallacies. What is fallacious in one reasoning system may not be fallacious under another. In this approach to validity and fallacy, I differ from the traditional approach to reasoning.

I have also suggested that the notion of validity should be defined relative to the individual or community critically evaluating knowledge claims. If so, a valid argument for one need not be valid for another, depending upon the particular rules of inference subscribed to.

Finally, I have suggested that a good argument be viewed as one that satisfies the criteria of validity, credibility of premises, confidence of conclusions, and global consistency. I have also made a distinction between explicit and implicit validity. I deviate from the traditional notion of sound arguments in these proposals as well.

---

[18] Note that the probabilistic deduction which makes use of the Probabilistic Modus Ponens (Rule IV, section 5.2.) is also truth preserving. In contrast, probabilistic deductive systems that sanction the inference of absolute conclusions from probabilistic premises (called statistical syllogism) lack truth preservation.

[19] My unhappiness with the traditional textbook classification is that there is no single set of rules of inference that characterize what they call 'inductive reasoning". Anything that does not fit the description of classical deduction is treated as induction in this approach.