

A Model for Integrating Fixed-, Random-, and Mixed-effects Meta-analyses into Structural Equation Modeling

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Introduction




- ⌘ Meta-analysis is a statistical approach to address:
 - ☑ whether or not the effect sizes are homogeneous;
 - ☑ what the overall effect size is; and
 - ☑ how the heterogeneity of the effect sizes may be explained by the moderators and/or by the random-effects models.



⌘ Structural equation modeling (SEM) is a general statistical model for:

- ☒ ANOVA to MANCOVA;
- ☒ Regression analysis;
- ☒ Path analysis;
- ☒ Confirmatory factor analysis;
- ☒ Latent growth modeling;
- ☒ Multilevel models; and
- ☒ More...

- 
- ⌘ These two statistical techniques have their own traditions, terminologies and software packages;
 - ⌘ They are treated as two unrelated topics in the literature:
 - ⊗ Meta-analysis: 27,000 citations
 - ⊗ SEM: 9,000 citations
 - ⊗ Meta-analysis and SEM: less than 60 citations

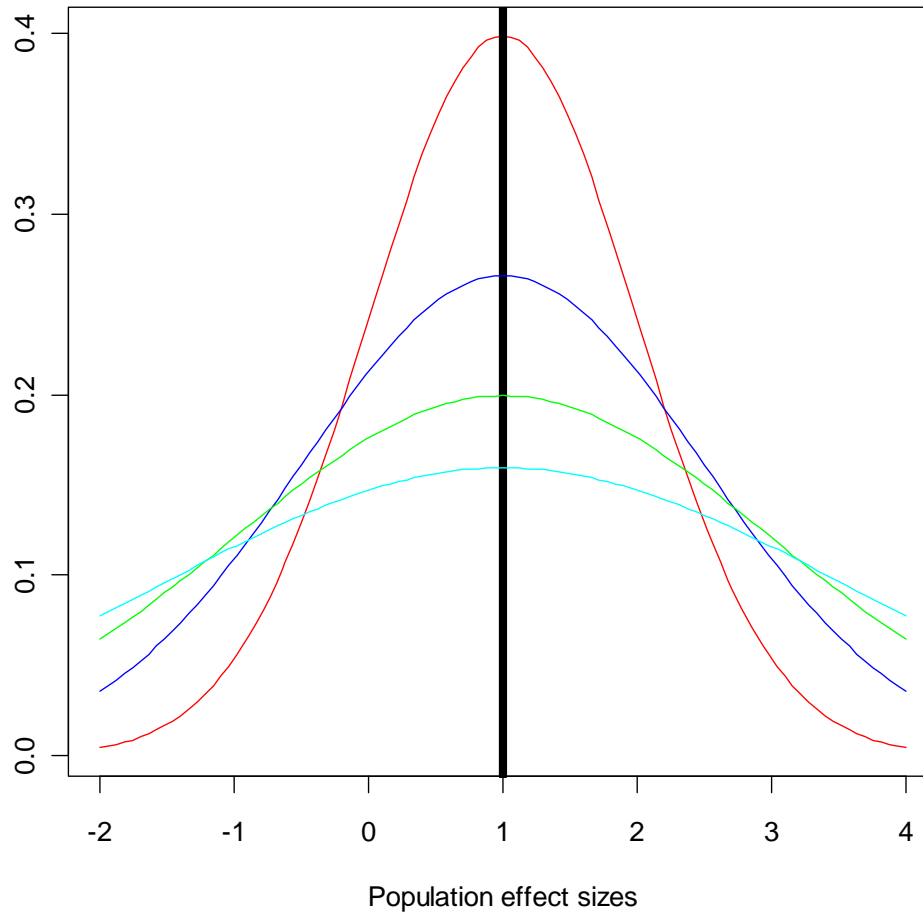
Objectives of this presentation



- ⌘ Propose a model to integrate fixed-, random-, and mixed-effect meta-analyses into SEM (Cheung, in press);
- ⌘ Demonstrate the advantages of using the SEM-based meta-analysis.

Cheung, M.W.L. (in press). A model for integrating fixed-, random-, and mixed-effects meta-analyses into structural equation modeling. *Psychological Methods*.

Fixed-effects model



Fixed-effects models

⌘ Model without any moderator

- ☑ Let y_i and β_i be the sample and population effect sizes in the i th study
- ☑ The sampling variance of e_i is usually assumed to be fixed and known

$$y_i = \beta_{\text{Fixed}} + e_i$$

$$e_i \sim N(0, \sigma_i^2)$$

- ⌘ The weighted mean and its sampling variance are obtained by using $w_i = 1/\sigma_i^2$ as the weight:

$$\hat{\beta}_{\text{Fixed}} = \frac{\sum_{i=1}^k w_i y_i}{\sum_{i=1}^k w_i}$$

$$s_{\text{Fixed}}^2 = 1 / \sum_{i=1}^k w_i$$

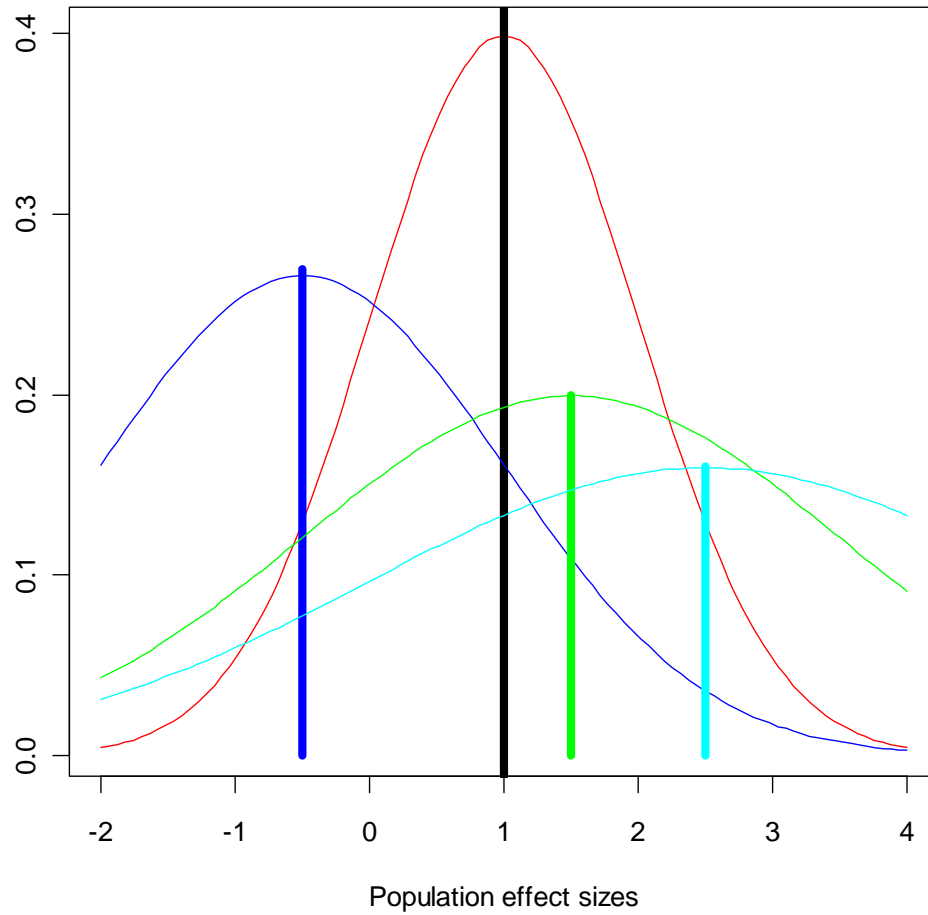


⌘ Model with a moderator

$$y_i = \beta_0 + \beta_1 x_{1i} + e_i$$

- ⌘ The parameter estimates and their sampling variance may be obtained by weighted least squares (Hedges & Olkin, 1985)

Random-effects model




Random-effects models

⌘ The model for the random-effects model is

$$y_i = \beta_{\text{Random}} + u_i + e_i$$

⌘ A common variance component estimator (DerSimonian & Laird, 1986) is

$$\hat{\tau}_{DL}^2 = \frac{Q - (k - 1)}{c} \quad c = \sum_{i=1}^k w_i - \left(\sum_{i=1}^k w_i^2 \right) / \left(\sum_{i=1}^k w_i \right)$$



⌘ Once the variance component is estimated, the weighted effect size can be estimated by using $w_i = 1/(\sigma^2_i + \tau^2)$ as the new weight

$$\hat{\beta}_{\text{Random}} = \frac{\sum_{i=1}^k \tilde{w}_i y_i}{\sum_{i=1}^k \tilde{w}_i} \quad s_{\text{Random}}^2 = 1 / \sum_{i=1}^k \tilde{w}_i$$



⌘ Model with a moderator

$$y_i = \beta_0 + \beta_1 x_{1i} + u_i + e_i$$

- ⌘ Multilevel models may be used to estimate the mixed-effects meta-analysis (Raudenbush & Bryk, 2002);
- ⌘ It is clear that the previous models are all special cases of the mixed-effects model.

Raudenbush, S. W., & Bryk, A. S. (2002). *Hierarchical linear models: Applications and data analysis methods (2nd Ed.)*. Thousand Oaks, CA: Sage Publications.

Major problems of fitting meta-analytic models in SEM



- ⌘ Data in a meta-analysis are assumed to be distributed with known variances;
- ⌘ Data in a SEM are usually assumed distributed with a common variance.

Transforming data to achieve identically distributed errors

⌘ We may transform the data in a meta-analysis such that the residuals are distributed with the same variance: 1.0

⌘ Recall that: $e_i \sim N(0, \sigma_i^2)$

$$W^{1/2} = \text{diag}[1/\sigma_1, 1/\sigma_2, \dots, 1/\sigma_k]$$

$$W^{1/2} y_i = W^{1/2} \beta_0 + W^{1/2} x_{1i} \beta_1 + W^{1/2} I_k u + W^{1/2} e$$

$$y_i^* = 1^* \beta_0 + x_{1i}^* \beta_1 + I_k^* u + e_i^* \quad e_i^* \sim N(0, 1)$$

SEM-based meta-analysis



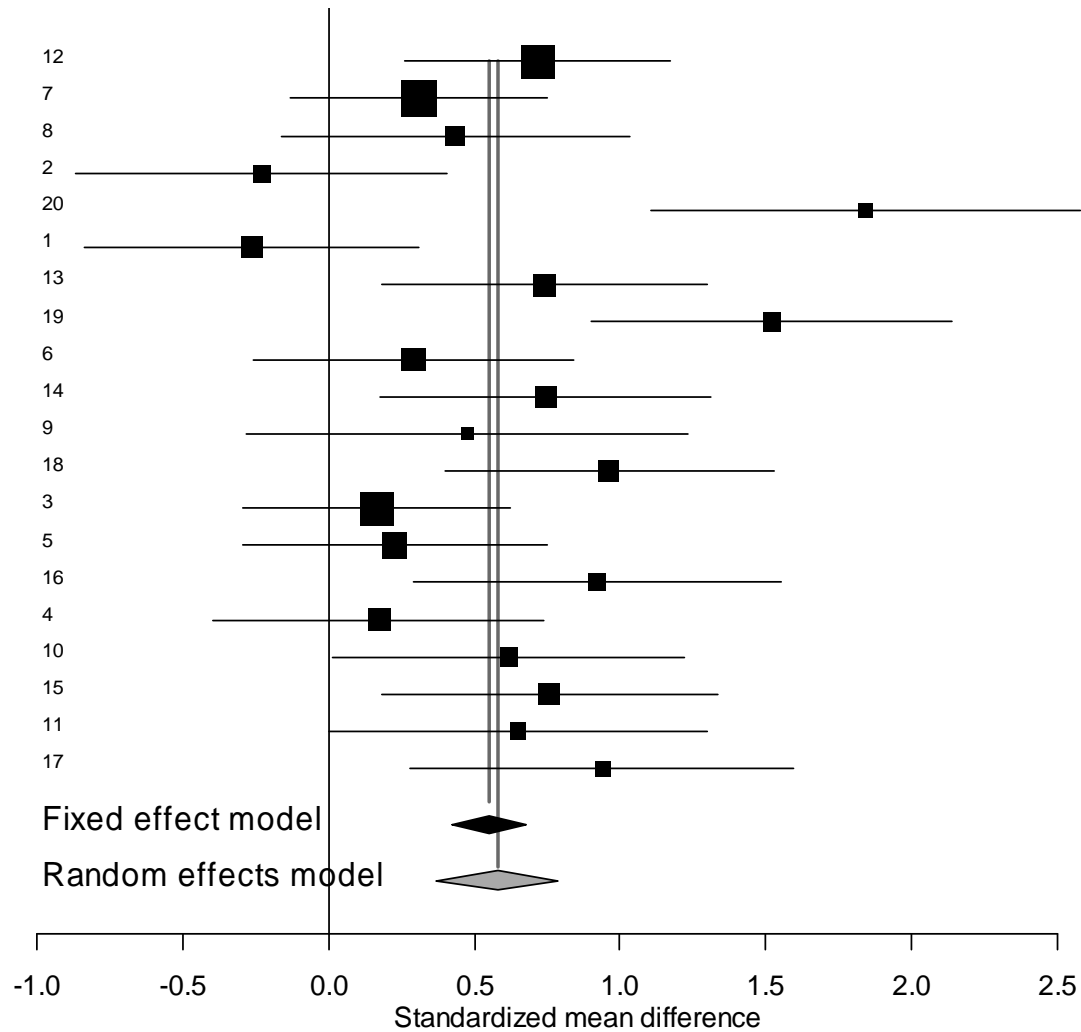
- ⌘ After the transformation, studies in a meta-analysis can be treated as subjects in a structural equation model.
- ⌘ Many powerful techniques in SEM are available to meta-analysts!

An illustration



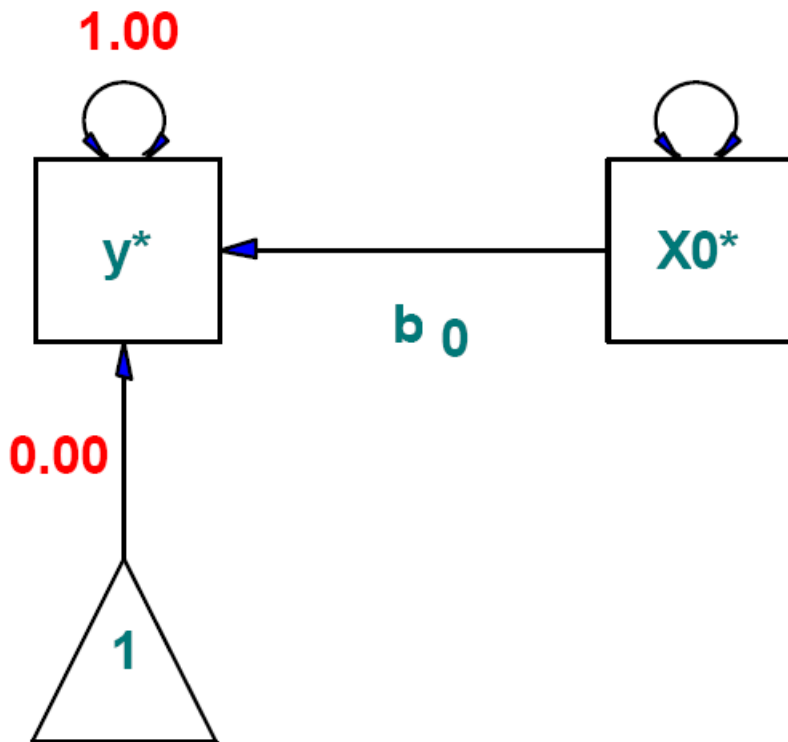
- ⌘ A summary of 20 simulated studies reported in Hox (2002) is listed in the next table.
- ⌘ Effect size: Hedges's d
- ⌘ Moderator: duration of the experimental intervention in terms of *weeks*

Plot of the studies



Study	d	var(d)	weeks	$W^{0.5}$	d^*	Intercept*	weeks*
1	-0.264	0.086	3	3.41	-0.9002	3.41	10.2299
2	-0.23	0.106	1	3.0715	-0.7064	3.0715	3.0715
3	0.166	0.055	2	4.264	0.7078	4.264	8.528
4	0.173	0.084	4	3.4503	0.5969	3.4503	13.8013
5	0.225	0.071	3	3.7529	0.8444	3.7529	11.2588
6	0.291	0.078	6	3.5806	1.0419	3.5806	21.4834
7	0.309	0.051	7	4.4281	1.3683	4.4281	30.9965
8	0.435	0.093	9	3.2791	1.4264	3.2791	29.5122
9	0.476	0.149	3	2.5906	1.2331	2.5906	7.7719
10	0.617	0.095	6	3.2444	2.0018	3.2444	19.4666
11	0.651	0.11	6	3.0151	1.9628	3.0151	18.0907
12	0.718	0.054	7	4.3033	3.0898	4.3033	30.1232
13	0.74	0.081	9	3.5136	2.6001	3.5136	31.6228
14	0.745	0.084	5	3.4503	2.5705	3.4503	17.2516
15	0.758	0.087	6	3.3903	2.5699	3.3903	20.3419
16	0.922	0.103	5	3.1159	2.8728	3.1159	15.5794
17	0.938	0.113	5	2.9748	2.7904	2.9748	14.8741
18	0.962	0.083	7	3.4711	3.3392	3.4711	24.2974
19	1.522	0.1	9	3.1623	4.813	3.1623	28.4605
20	1.844	0.141	9	2.6631	4.9108	2.6631	23.9681

SEM for a fixed-effects meta-analysis without any covariate



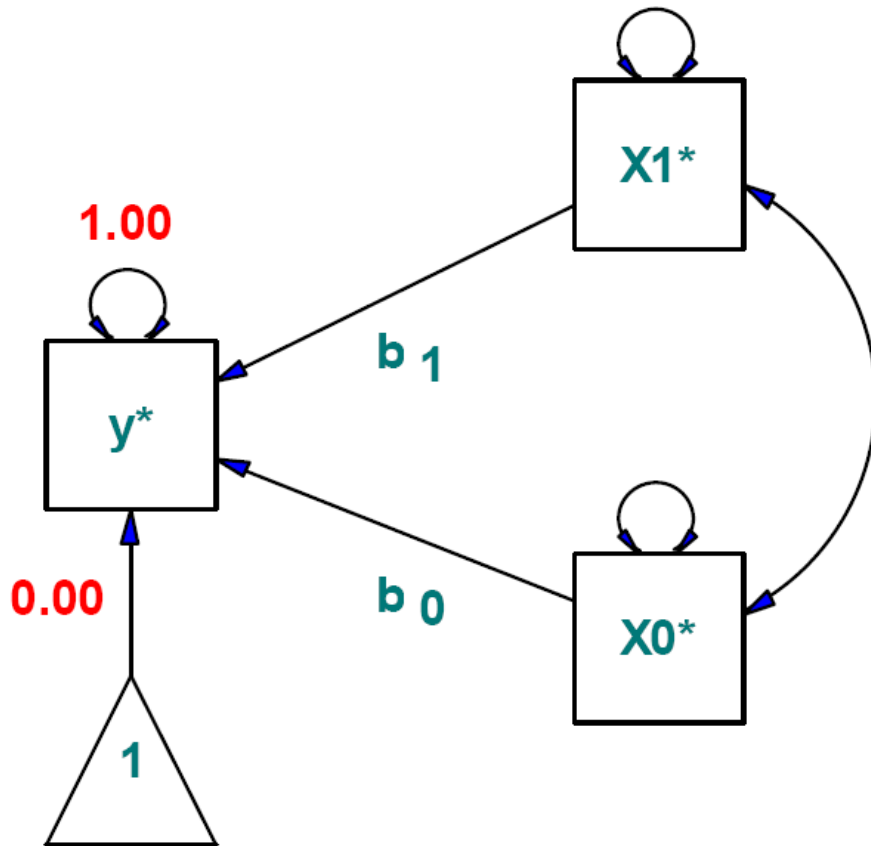
Key points:

1. Error variance on y^* is fixed at 1;
2. The intercept is represented by $X0^*$

Results:

1. The weighted effect size b_0 is 0.550 (0.065).

SEM for a fixed-effects meta-analysis with a covariate



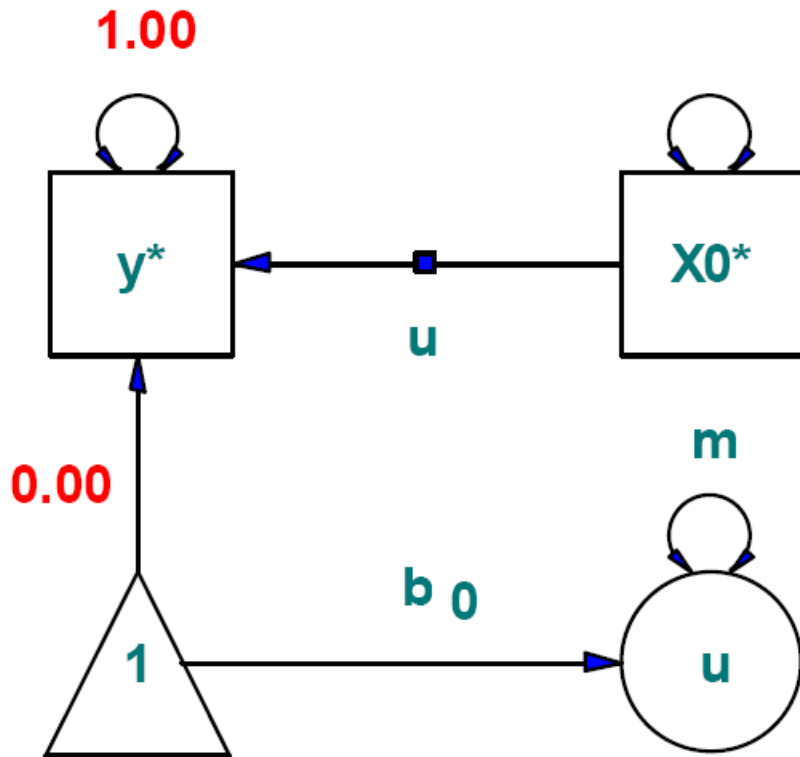
Key points:

1. $X0^*$: intercept
2. $X1^*$: moderator (*weeks*)

Results:

1. The intercept and the slope (for *weeks*) are -0.204 (0.170) and 0.135 (0.028).
2. When “weeks” increases one unit, the effect size increases by 0.135 unit.

SEM for a random-effects meta-analysis without any covariate



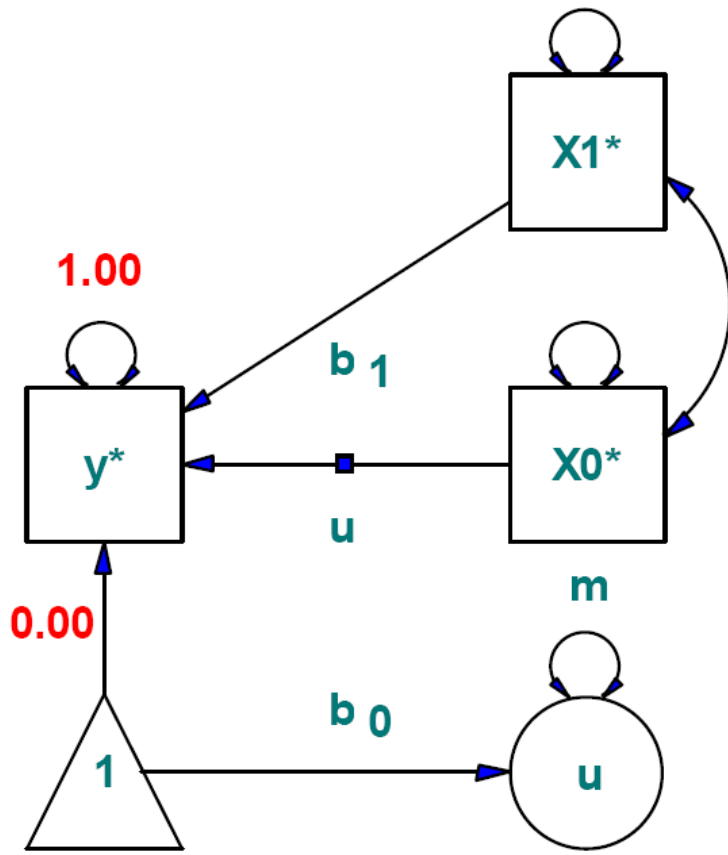
Key points:

1. u is the random effect
2. Random slope analysis is required (Mehta & Neale, 2005)
3. b_0 is the weighted effect size
4. m is the variance component (the amount of heterogeneity).

Results:

1. The weighted effect size b_0 is 0.579 (0.107)
2. The variance component m is 0.132

SEM for a mixed-effects meta-analysis with a covariate



Results:

1. The intercept and the slope are -0.214 (0.171) and 0.139 (0.036).
2. The residual variance component is 0.023.

Advantages and extensions of the SEM-based meta-analysis



- ⌘ Here are some of the advantages of using the SEM-based meta-analysis (see Cheung, in press for the details):
 - ☑ Handling missing covariates with maximum likelihood estimation method;
 - ☑ Robust standard error;
 - ☑ Constructing confidence intervals on parameter estimates and heterogeneity indices; and
 - ☑ Addressing heterogeneity with mixture models.



Thank you!

Paper and Mplus codes for the demonstration are available at:

<http://courses.nus.edu.sg/course/psycwlm/internet/publications.html>