(1) (Welty, Rorrer, Foster, 6th Edition International Student Version 14.7)
A 0.20-m-thick brick wall (k=1.3W/m•K) separated the combustion zone of a furnace from its surroundings at 25°C. For an outside wall surface temperature of 100°C, with a convective heat transfer coefficient of 18 W/m²•K, what will be the inside wall surface temperature at steady-state conditions?

Solution :
\[
\frac{q}{A} = \frac{k}{L} \Delta T_w = h(T_s - T_{surrounding})
\]

\[
\Delta T_w = \frac{h(T_s - T_{surrounding}) \times L}{k}
\]

\[
= \frac{(18W/m.k)(75k)(20 \times 10^{-2}m)}{1.3(W/m.k)}
\]

\[
= 207.7k
\]

\[T_{inside} = 100 + \Delta T_w = 307.7^\circ C\]

(2) (Welty, Rorrer, Foster, 6th Edition International Student Version 14.10)
The cross section of a storm window is shown in the sketch. How much heat will be lost through a window measuring 1.83m by 3.66 m on a cold day when the inside and outside air temperatures are, respectively, 295K and 250K? Convective coefficients on the inside and outside surfaces of a window are 20 W/m²•K and 15 W/m²•K, respectively. What temperature drop will exist across each of the glass panes? What will be the average temperature of the air between the glass panes?

Solution:
\[
q = \frac{\Delta T}{\sum R} = \text{?}
\]

\[
R_i = \frac{1}{(20)(1.83)(3.66)} = 7.46 \times 10^{-3}
\]

\[
R_{GL} = \frac{0.0032}{(0.78)(1.83)(3.66)} = 6.125 \times 10^{-4}
\]

\[
R_{AS} = \frac{0.008}{(0.0245)(1.83)(3.66)} = 0.0488
\]

\[
R_0 = \frac{1}{(15)(1.83)(3.66)} = 9.95 \times 10^{-3}
\]

\[
\sum R = 0.06744
\]

\[
q = \frac{\Delta T}{\sum R} = \frac{45}{0.06744} = 667 W
\]
Compare the heat lost through the storm window described in problem (2) with the same conditions existing except that the window is a single pane of glass 0.32 cm thick.

Solution:

For single pane of glass only

\[ \sum R = R_i + R_{gl} + R_o = 0.018 \]

\[ q = \frac{\Delta T}{\sum R} = \frac{45}{0.018} = 2500W \]

(4) PROBLEM 2.3 (ID)

A hot water pipe with outside radius \( r_1 \) has a temperature \( T_1 \). A thick insulation applied to reduce the heat loss has an outer radius of \( r_2 \) and temperature of \( T_2 \). On \( T-r \) coordinates, sketch the temperature distribution in the insulation for one-dimensional, steady state heat transfer with constant properties. Give a brief explanation justifying the shape of your curve.

KNOWN: Hot water pipe covered with thick layer of insulation.
FIND: Sketch temperature distribution and give brief explanation to justify shape.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional (radial) conduction, (3) No internal heat generation, (4) Insulation has uniform properties independent of temperature and position.

ANALYSIS: Fourier’s law, Eq. 2.1, for this one-dimensional (cylindrical) radial system has the form

\[ q_r = -kA \frac{dT}{dr} = -k(2\pi r)e \frac{dT}{dr} \]
where $A_T = 2\pi \ell$ and $\ell$ is the axial length of the pipe-insulation system. Recognize that for steady-state conditions with no internal heat generation, an energy balance on the system requires $E_{\text{in}} = E_{\text{out}}$ since $E_g = E_{\text{st}} = 0$. Hence

$$q_T = \text{Constant.}$$

That is, $q_T$ is independent of radius ($r$). Since the thermal conductivity is also constant, it follows that

$$r \frac{dT}{dr} = \text{Constant.}$$

This relation requires that the product of the radial temperature gradient, $dT/dr$, and the radius, $r$, remains constant throughout the insulation. For our situation, the temperature distribution must appear as shown in the sketch.

**COMMENTS:** (1) Note that, while $q_T$ is a constant and independent of $r$, $q_T''$ is not a constant. How does $q_T''(r)$ vary with $r$? (2) Recognize that the radial temperature gradient, $dT/dr$, decreases with increasing radius.

(5) **PROBLEM 2.4 (ID)**

A spherical shell with inner radius $r_1$ and outer radius $r_2$ has surface temperature $T_1$ and $T_2$ respectively, where $T_1 > T_2$. Sketch the temperature distribution on $T$-$r$ coordinates, assuming steady state, one dimensional conduction with constant properties. Briefly justify the shape of your curve.

**KNOWN:** A spherical shell with prescribed geometry and surface temperatures.

**FIND:** Sketch temperature distribution and explain shape of the curve.

**SCHEMATIC:**
**ASSUMPTIONS:** (1) Steady-state conditions, (2) One-dimensional conduction in radial (spherical coordinates) direction, (3) No internal generation, (4) Constant properties.

**ANALYSIS:** Fourier’s law, Eq. 2.1, for this one-dimensional, radial (spherical coordinate) system has the form

\[
q_r = -kA_r \frac{dT}{dr} = -k \left( 4\pi r^2 \right) \frac{dT}{dr}
\]

where \(A_r\) is the surface area of a sphere. For steady-state conditions, an energy balance on the system yields \(E_{\text{in}} = E_{\text{out}}\), since \(E_g = E_{\text{st}} = 0\). Hence,

\[
q_{\text{in}} = q_{\text{out}} = q_r \neq q_r(r).
\]

That is, \(q_r\) is a constant, independent of the radial coordinate. Since the thermal conductivity is constant, it follows that

\[
r^2 \left( \frac{dT}{dr} \right) = \text{Constant}.
\]

This relation requires that the product of the radial temperature gradient, \(dT/dr\), and the radius squared, \(r^2\), remains constant throughout the shell. Hence, the temperature distribution appears as shown in the sketch.

**COMMENTS:** Note that, for the above conditions, \(q_r \neq q_r(r)\); that is, \(q_r\) is everywhere constant. How does \(q_r^r\) vary as a function of radius?