EXERCISE 8.1:
Design a positive pressure dilute-phase pneumatic transport system to carry 500 kg/hr of a powder of particle density 1800 kg/m³ and mean particle size 150 μm across a horizontal distance of 100 metres and a vertical distance of 20 metres using ambient air. Assume that the pipe is smooth, that four 90° bends are required and that the allowable pressure loss is 0.7 bar.

SOLUTION TO EXERCISE 8.1:
Design in this case means determine the pipe size and air flowrate which would give a total system pressure loss near to, but not exceeding, the allowable pressure loss. The design procedure requires trial and error calculations. Pipes are available in fixed sizes and so the procedure is to select a pipe size and determine the saltation velocity from Text-Equation 8.3. Saltation velocity is important since, in any system with horizontal and vertical pipelines, the saltation velocity is always greater than the choking velocity - so if we avoid saltation, we avoid choking. The system pressure loss is then calculated at a superficial gas velocity equal to 1.5 times the saltation velocity (this gives a reasonable safety margin bearing in mind the accuracy of the correlation in Text-Equation 8.3). The calculated system pressure loss is then compared with the allowable pressure loss. The pipe size selected may then be altered and the above procedure repeated until the calculated pressure loss matches that allowed.

**Step 1 Selection of pipe size:** Select 50 mm internal diameter pipe.

**Step 2 Determine gas velocity**

Use the Rizk correlation of Text-Equation (8.3) to estimate the saltation velocity, \( U_{\text{SALT}} \). Text-Equation (8.3) rearranged becomes:

\[
U_{\text{SALT}} = \left( \frac{4M_p 10^6 \frac{\alpha}{\beta} \frac{D}{D_f} \left( D_f^{-2} \right)}{\pi \rho_f} \right)^{\frac{1}{\beta+1}}
\]

where \( \alpha = 1440x + 1.96 \) and \( \beta = 1100x + 2.5 \).

In the present case \( \alpha = 2.176, \beta = 2.665 \) and \( U_{\text{SALT}} = 9.21 \text{ m/s} \).

Therefore, superficial gas velocity, \( U = 1.5 \times 9.211 \text{ m/s} = 13.82 \text{ m/s} \).
**Step 3 Pressure loss calculations**

*a) Horizontal Sections*

Starting with Text-Equation 8.15 and expression for the total pressure loss in the horizontal sections of the transport line may be generated. We will assume that all the initial acceleration of the solids and the gas take place in the horizontal sections and so terms 1 and 2 are required. For term 3 the Fanning friction Equation is used assuming that the pressure loss due to gas/wall friction is independent of the presence of solids. For term 4 we employ the Hinkle correlation (Text-Equation 8.17). Terms 5 and 6 became zero as \( \theta = 0 \) for horizontal pipe. Thus, the pressure loss, \( \Delta P_H \), in the horizontal sections of the transport line is given by:

\[
\Delta P_H = \frac{\rho_p \varepsilon_H U_{H}^2}{2} + \frac{\rho_p (1 - \varepsilon_H) U^2}{2} + \frac{2f_p \rho_p U^2 L_H}{D} + \frac{2f_p \rho_p (1 - \varepsilon_H) U_{pH}^2 L_H}{D}
\]

where the subscript \( H \) refers to the values specific to the horizontal sections.

To use this Equation we need to know \( \varepsilon_H \), \( U_{H} \) and \( U_{pH} \). Hinkle's correlation gives us \( U_{pH} \):

\[
U_{pH} = U (1 - 0.0638 \times 0.3 \times 0.5) = 11.15 \text{ m/s}
\]

From continuity, \( G = \rho_p (1 - \varepsilon_H) U_{pH} \).

Solids flux, \( G = M_p / A = \frac{500}{3600} \times \frac{1}{\pi (0.05)^2} = 70.73 \text{ kg/m}^2 \cdot \text{s} \)

thus \( \varepsilon_H = 1 - \frac{G}{\rho_p U_{pH}} = 0.9965 \)

and \( U_{H} = \frac{U}{\varepsilon_H} = \frac{13.82}{0.9965} = 13.87 \text{ m/s} \)

Friction factor \( f_p \) is found from Text-Equation (8.19) with \( C_D \) estimated at the relative velocity (\( U_{pH} - U_{pH} \)), using the approximate correlations given below, (or by using an appropriate \( C_D \) versus \( Re \) chart [see Chapter 2])

- \( Re_p < 1 \) : \( C_D = 24 / Re_p \)
- \( 1 < Re_p < 500 \) : \( C_D = 18.5 Re_p^{-0.6} \)
- \( 500 < Re_p < 2 \times 10^5 \) : \( C_D = 0.44 \)
Thus, for flow in the horizontal sections, $Re_p = \frac{\rho f (U_{in} - U_{out}) \times k}{\mu}$

For ambient air, $\rho_f = 1.2$ kg/m$^3$ and $\mu = 18.4 \times 10^{-6}$ Pas, giving

$$Re_p = \frac{150 \times 10^{-6} \times 1.2 \times (13.87 - 11.15)}{18.4 \times 10^{-6}} = 26.5$$

and so, using the approximate correlations above, $C_D = 18.5 Re^{-0.6} = 2.59$

Substituting $C_D = 2.59$ in Text-Equation 8.19 we have:

$$f_p = \frac{3}{8} \times \frac{1.2}{1800} \times 2.59 \times \frac{0.050}{150 \times 10^{-6}} \left\{ \frac{13.87 - 11.15}{11.15} \right\}^2 = 0.01277$$

To estimate the gas friction factor we use the Blasius correlation for smooth pipes, $f_g = 0.079 \times Re^{-0.25}$. The Reynolds number calculated based on the superficial gas velocity:

$$Re = \frac{0.05 \times 1.2 \times 13.82}{18.4 \times 10^{-6}} = 45065$$, which gives $f_g = 0.0054$.

Thus the components of the pressure loss in the horizontal pipe from Text-Equation 8.15 are:

Term 1 (gas acceleration):

$$= \frac{\rho_f \varepsilon H U_{H}^2}{2} = \frac{1.2 \times 0.9965 \times 13.87^2}{2} = 114.9 \text{ Pa.}$$

Term 2 (solids acceleration):

$$= \frac{\rho_p (1 - \varepsilon_p) U_{pH}^2}{2} = \frac{1800 \times (1 - 0.9965) \times 11.15^2}{2} = 394.4 \text{ Pa.}$$

Term 3 (gas friction):

$$= \frac{2f_g \rho_f U_{H}^2 L_{H}}{D} = \frac{2 \times 0.0054 \times 1.2 \times 13.82^2 \times 100}{0.05} = 4968 \text{ Pa.}$$

Term 4 (solids friction):

$$= \frac{2f_p \rho_p (1 - \varepsilon_p) U_{pH}^2 L}{D} = \frac{2 \times 0.01276 \times 1800 \times (1 - 0.9965) \times 11.15^2 \times 100}{0.05} = 40273 \text{ Pa.}$$

This gives $\Delta p_{H} = 45751 \text{ Pa.}$
b) Vertical Sections

Starting again with Text-Equation 8.15, the general pressure loss Equation, an expression for the total pressure loss in the vertical section may be derived. Since the initial acceleration of solids and gas was assumed to take place in the horizontal sections, terms 1 and 2 become zero. The Fanning friction Equation is used to estimate the pressure loss due to gas-to-wall friction (term 3) assuming solids have negligible effect on this pressure loss. For term 4 the modified Konno and Saito correlation (Text-Equation 8.16) is used. For vertical transport $\theta$ is 90° in terms 5 and 6.

Thus, the pressure loss, $\Delta p_v$, in the vertical sections of the transport line is given by:

$$\Delta p_v = \frac{2f_g \rho_f U^2 L_v}{D} + 0.057 \rho_f \rho_s (1 - \varepsilon_v) g L_v + \rho_f \varepsilon_v g L_v$$

where subscript $v$ refers to values specific to the vertical sections.

To use this Equation we need to calculate the voidage of the suspension in the vertical pipe line $\varepsilon_v$:

Assuming particles behave as individuals, then slip velocity is equal to single particle terminal velocity, $U_T$ (also noting that the superficial gas velocity in both horizontal and vertical section is the same and equal to $U$)

$$i.e. \quad U_{pv} = \frac{U}{\varepsilon_v} - U_T$$

continuity gives particle mass flux, $G = \rho_p (1 - \varepsilon_v) U_{pv}$

Combining these Equations gives a quadratic in $\varepsilon_v$ which has only one possible root.

$$\varepsilon_v^2 U_T - \left[ U_T + U - \frac{G}{\rho_p} \right] \varepsilon_v + U = 0$$

The single particle terminal velocity, $U_T$ may be estimated as shown in Chapter 2, giving $U_T = 0.715$ m/s assuming the particles are spherical.

And so, solving the quadratic Equation, $\varepsilon_v = 0.9970$

The components of the pressure loss in the vertical pipe are therefore:
Term 3 (gas friction): \[ \frac{2 \ell_\ell_p \rho \nu U^2 L_v}{D} = \frac{2 \times 0.0054 \times 1.2 \times 13.82^2 \times 20}{0.05} = 993.7 \text{ Pa.} \]

Term 4 (solids friction):
\[ = 0.057 \times G \sqrt{\frac{g}{D}} = 0.057 \times 70.73 \times 20 \times \sqrt{\frac{9.81}{0.05}} = 1129.4 \text{ Pa} \]

Term 5 (solids gravitational head):
\[ = \rho_p (1 - \varepsilon_v) g L_v = 1800 \times (1 - 0.9970) \times 9.81 \times 20 = 1055.8 \text{ Pa} \]

Term 6 (gas gravitational head):
\[ = \rho_g \varepsilon_g g L_v = 1.2 \times 0.9970 \times 9.81 \times 20 = 234.7 \text{ Pa} \]

and thus total pressure loss across vertical sections, \( \Delta p_v = 3414 \text{ Pa} \)

c) Bends
The pressure loss across each 90 degree bend is taken to be equivalent to that across 7.5 m of vertical pipe.
Pressure loss per metre of vertical pipe = \( \frac{\Delta p_v}{L_v} = 170.7 \text{ Pa} / \text{m} \)

Therefore, pressure loss across four 90° bends
\[ = 4 \times 7.5 \times 170.7 \text{ Pa} \]
\[ = 5120.4 \text{ Pa} \]

And so,
\[
\begin{pmatrix}
\text{total pressure loss} \\
\text{loss across vertical sections} \\
\text{loss across horizontal sections} \\
\text{loss across bends}
\end{pmatrix}
= \begin{pmatrix} 3413.6 \\ 45751.6 \\ 5120.4 \end{pmatrix} \text{ Pa}
= 0.543 \text{ bar}
\]

Step 4 Compare calculated and allowable pressure losses

The allowable system pressure loss is 0.7 bar and so we may select a smaller pipe size and repeat the above calculation procedure. The table below gives the results for a range of pipe sizes.
<table>
<thead>
<tr>
<th>Pipe inside diameter (mm)</th>
<th>Total System Pressure Loss (bar)</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>0.543</td>
</tr>
<tr>
<td>40</td>
<td>0.857</td>
</tr>
</tbody>
</table>

In this case we would select 50 mm pipe which gives a total system pressure loss of 0.543 bar. The design details for this selection are given below:

Pipe size: 50 mm inside diameter
Air flowrate = 0.027 m³/s
Air superficial velocity = 13.82 m/s
Saltation velocity = 9.21 m/s
Solids loading = 4.26 kg solid/kg air
Total system pressure loss = 0.543 bar

**EXERCISE 8.2:**
It is required to use an existing 50 mm inside diameter vertical smooth pipe as lift line to transfer 2000 kg/hr of sand of mean particle size 270 μm and particle density 2500 kg/m³ to a process 50 metres above the solids feed point. A blower is available which is capable of delivering 60 m³/hr of ambient air at a pressure of 0.3 bar. Will the system operate as required?

**SOLUTION TO EXERCISE 8.2:**
To test whether the system will operate, we will first check that the air volume flow rate is satisfactory:

The superficial gas velocity in the lift line must exceed the predicted choking velocity by a reasonable safety margin. The choking velocity is predicted using Text-Equation 8.1 and 8.2.

\[
\frac{U_{CH}}{e_{CH}} - U_T = \frac{G}{\rho_p (1 - e_{CH})} \quad \text{(Text-Equation 8.1)}
\]

\[
\rho_T^{0.77} = \frac{2250D(e_{CH}^{-1.7} - 1)}{\left[\frac{U_{CH}}{e_{CH}} - U_T\right]^2} \quad \text{(Text-Equation 8.2)}
\]

The single particle terminal velocity, \( U_T \) may be estimated as shown in Chapter 2, giving \( U_T = 1.77 \text{ m/s} \) (assuming the particles are spherical).
Solids flux, \( G = \frac{M_p}{A} = \frac{2000}{3600} \times \frac{1}{\pi \left(0.05\right)^2} = 282.9 \ \text{kg/m}^2 \cdot \text{s} \)

Substituting Text-Equation 8.1 into Text-Equation 8.2 gives:

\[
\rho_f^{0.77} = \frac{2250D\left(\varepsilon_{CH}^{-6} - 1\right)\rho_f^{2}\left(1 - \varepsilon_{CH}\right)^2}{G^2}
\]

which can be solved by trial and error to give \( \varepsilon_{CH} = 0.9705 \).

Substituting back into Text-Equation 8.1 gives choking velocity \( U_{CH} = 5.446 \ \text{m/s} \).

Actual maximum volume flow rate available at the maximum pressure is 60 m\(^3\)/h, which in a 50 mm diameter pipe gives a superficial gas velocity of 8.49 m/s. Operating at this superficial gas velocity would give us a 56% safety margin over the predicted choking velocity \( U = U_{CH} \times 1.56 \), which is acceptable.

The next step is to calculate the lift line pressure loss at this gas flow rate and compare it with the available blower pressure at this flow rate.

Starting with Text-Equation 8.15, the general pressure loss Equation, an expression for the total pressure loss in the vertical lift line may be derived. Initial acceleration of solids and gas must be taken into account and so terms 1 and 2 are included. The Fanning friction Equation is used to estimate the pressure loss due to gas-to-wall friction (term 3) assuming solids have negligible effect on this pressure loss. For term 4 the modified Komno and Saito correlation (Text-Equation 8.16) is used. For vertical transport \( \theta \) is 90\(^\circ\) in terms 5 and 6.

Thus, the pressure loss, \( \Delta p_v \), in the vertical sections of the transport line is given by:

\[
\Delta p_v = \frac{\rho_p \varepsilon_v U_{pv}^2}{2} + \frac{\rho_f \left(1 - \varepsilon_v\right) U_{pv}^2}{2} + \frac{2 \rho_f \rho_f U_{pv}^2 L_v}{D} + 0.057 L \sqrt{\frac{g}{D}} + \rho_p \left(1 - \varepsilon_v\right) g L_v + \rho_f \varepsilon_v g L_v
\]

To use this Equation we need to calculate the voidage of the suspension in the vertical pipe line \( \varepsilon_v \).

Assuming particles behave as individuals, then slip velocity is equal to single particle terminal velocity, \( U_T \).

\[
i.e. \quad U_{pv} = \frac{U}{\varepsilon_v} - U_T
\]

continuity gives particle mass flux, \( G = \rho_p \left(1 - \varepsilon_v\right) U_{pv} \)
Combining these Equations gives a quadratic in $\varepsilon_v$ which has only one possible root.

\[ \varepsilon_v^2 U_T - \left[ U_T + U + \frac{G}{\rho_p} \right] \varepsilon_v + U = 0 \]

The single particle terminal velocity, $U_T$, was found above to be 1.77 m/s. 
And so, solving the quadratic Equation, $\varepsilon_v = 0.9835$ and actual gas velocity,

\[ \frac{U}{\varepsilon_v} = \frac{8.49}{0.9835} = 8.63 \text{ m/s} \]

Then actual solids velocity, $U_{pv} = U_{fv} - U_T = 8.63 - 1.77 = 6.86 \text{ m/s}$

The components of the pressure loss in the vertical pipe are therefore:

**Term 1 (gas acceleration):**

\[ \frac{\rho_g \varepsilon_v U_{pv}^2}{2} = \frac{1.2 \times 0.9835 \times 8.63^2}{2} = 43.9 \text{ Pa.} \]

**Term 2 (solids acceleration):**

\[ \frac{\rho_p (1 - \varepsilon_v) U_{pv}^2}{2} = \frac{2500 \times (1 - 0.9835) \times 6.86^2}{2} = 970.5 \text{ Pa.} \]

**Term 3 (gas friction):**

Estimate the gas friction factor using the Blasius correlation for smooth pipes, $f_g = 0.079 \times Re^{-0.25}$. The Reynolds number calculated based on the superficial gas velocity:

\[ Re = \frac{0.05 \times 1.2 \times 8.49}{18.4 \times 10^{-6}} = 27679 \text{, which gives } f_g = 0.0061. \]

Then, term 3:

\[ \frac{2f_g \rho_g U_{pv}^2 L_v}{D} = \frac{2 \times 0.0061 \times 1.2 \times 8.49^2 \times 50}{0.05} = 1059.1 \text{ Pa.} \]

**Term 4 (solids friction):**

\[ = 0.057 \times GL \sqrt{\frac{g}{D}} = 0.057 \times 282.9 \times 50 \times \sqrt{\frac{9.81}{0.05}} = 11293.6 \text{ Pa.} \]

**Term 5 (solids gravitational head):**

\[ = \rho_p (1 - \varepsilon_v) g L_v = 2500 \times (1 - 0.9835) \times 9.81 \times 50 = 20226 \text{ Pa.} \]
Term 6 (gas gravitational head): \[ \rho_L \varepsilon_f g L_v = 1.2 \times 0.9835 \times 9.81 \times 50 = 579 \text{ Pa}. \]

and thus, total pressure loss across vertical sections, \( \Delta p_v = 33160 \text{ Pa} (0.332 \text{ bar}) \)

The available blower pressure at this maximum flow rate is 0.3 bar and so the lift line will not operate as required. Reducing the gas velocity safety margin will not help, since this will cause the line pressure loss to increase.