EXERCISE 7.1:
A packed bed of solid particles of density 2500 kg/m$^3$, occupies a depth of 1 m in a vessel of cross-sectional area 0.04 m$^2$. The mass of solids in the bed is 50 kg and the surface-volume mean diameter of the particles is 1 mm. A liquid of density 800 kg/m$^3$ and viscosity 0.002 Pas flows upwards through the bed.

(a) Calculate the voidage (volume fraction occupied by voids) of the bed.
(b) Calculate the pressure drop across the bed when the volume flow rate of liquid is 1.44 m$^3$/h.
(c) Calculate the pressure drop across the bed when it becomes fluidized.

SOLUTION TO EXERCISE 7.1:
(a) Bed voidage (volume fraction occupied by the voids) is calculated from Text-Equation 7.24:
\[ M = (1 - \varepsilon) \rho_p AH \]

Hence, voidage, \( \varepsilon = 1 - \frac{50}{2500 \times 0.04 \times 1} = 0.5 \)

(b) Pressure drop across the bed when the flow rate is 1.44 m$^3$/h:

Assume firstly that the bed is not fluidized at this flow rate. Estimate the pressure drop from the Ergun Equation (Text-Equation 7.3):
\[ \frac{(-\Delta p)}{H} = 150 \frac{(1-\varepsilon)^2}{\varepsilon^3} \frac{\mu U}{x_{sv}^2} + 1.75 \frac{(1-\varepsilon) \rho_f U^2}{\varepsilon^3} \frac{1}{x_{sv}} \]

Superficial liquid velocity, \( U = \frac{1.44}{0.04 \times 3600} = 0.01 \text{ m/s} \)

\( \mu = 0.002 \text{ Pa.s}; \; \varepsilon = 0.5; \; \rho_f = 800 \text{ kg/m}^3; \; H = 1.0 \text{ m}; \; x_{sv} = 10^{-3} \text{ m}. \)

Hence, \( (-\Delta p) = 6560 \text{ Pa} \)

(c) Check if the bed is fluidized: When fluidized, the apparent weight of the bed will be supported by the pressure difference. Hence (Text-Equation 7.2),
\[ \Delta p = H(1 - \varepsilon)(\rho_p - \rho_f) g \]
\((-\Delta p) = 1.0 \times (1 - 0.5) \times (2500 - 800) \times 9.81 = 8338.5 \text{ Pa}.\)

So the assumption in part (b) is correct and the answer to part (c) is 8338.5 Pa.

EXERCISE 7.2:
130 kg of uniform spherical particles with a diameter of 50 \(\mu\text{m}\) and particle density 1500 kg/m\(^3\) are fluidized by water (density 1000 kg/m\(^3\), viscosity 0.001 Pas.) in a circular bed of cross-sectional area 0.2 m\(^2\). The single particle terminal velocity of the particles is 0.68 mm/s and the voidage at incipient fluidization is known to be 0.47.

(a) Calculate the bed height at incipient fluidization.
(b) Calculate the mean bed voidage when the liquid flow rate is 2 \(\times\) 10\(^{-5}\) m\(^3\)/s.

SOLUTION TO EXERCISE 7.2:
(a) Bed height at incipient fluidization.

From Text-Equation 7.24: mass of solids in the bed, \(M = (1 - \varepsilon_{mf}) \rho_p A H_{mf}\)

Therefore, with \(M = 130\) kg, \(\varepsilon_{mf} = 0.47\), \(\rho_p = 1500\) kg/m\(^3\) and \(A = 0.2\) m\(^2\),

\[H_{mf} = \frac{130}{0.2 \times (1 - 0.47) \times 1500} = 0.818\ \text{m}\]

Bed height at incipient fluidization, \(H_{mf} = 0.818\) m.

(b) Bed height when liquid flow rate is 2 \(\times\) 10\(^{-5}\) m\(^3\)/s:

Use Richardson-Zaki equation (Equation 7.21), \(U = U_T \varepsilon^n\)

To determine exponent \(n\), calculate single particle Reynolds number, \(\text{Re}_p\) at \(U = U_T:\)

\[\text{Re}_p = \frac{U_T \rho_p \varepsilon}{\mu} = \frac{(0.68 \times 10^{-3}) \times 1000 \times (50 \times 10^{-6})}{0.001}\]

\[= 0.034, \text{ which is less than 0.3. Hence, } n = 4.65 \text{ (Text-Equation 7.22)}\]

Hence, applying the Richardson-Zaki equation, \(1 \times 10^{-4} = (0.68 \times 10^{-3}) \varepsilon^{4.65}\)
which gives, $\varepsilon = 0.6622$

hence, bed voidage at a liquid flow rate of $2 \times 10^{-5} \text{ m}^3/\text{s}$ is $\varepsilon = 0.6622$

**EXERCISE 7.5:**

12 kg of spherical resin particles of density 1200 kg/m$^3$ and uniform diameter 70 $\mu$m are fluidized by water (density 1000 kg/m$^3$ and viscosity 0.001 Pas.) in a vessel of diameter 0.3 m and form an expanded bed of height 0.25 m.

(a) Calculate the difference in pressure between the base and the top of the bed.
(b) If the flow rate of water is increased to 7 cm$^3$/s, what will be the resultant bed height and bed voidage (liquid volume fraction)? State and justify the major assumptions.

**SOLUTION TO EXERCISE 7.5:**

(a) The frictional pressure loss is given by the force balance over the fluidized bed

$$ (-\Delta p)A = \text{weight} - \text{upthrust} = Mg - M\frac{Pt}{\rho_p} g = Mg \left[1 - \frac{Pt}{\rho_p} \right]$$

Hence,

$$(-\Delta p) = 12 \times \left[1 - \frac{1000}{1200}\right] \times 9.81 \times \frac{\pi (0.3)^2}{4} = 277.5 \text{ Pa.}$$

Frictional pressure drop $(-\Delta p) = 277.5$ Pa.

However, the measured pressure drop across the bed will include the hydrostatic head of the liquid in the bed. Applying the mechanical energy equation between the bottom (1) and the top (2) of the fluidized bed:

$$\frac{Pt - P_2}{\rho_f g} + \frac{U_1^2 - U_2^2}{2g} + (z_1 - z_2) = \text{friction head loss} = \frac{277.5}{\rho_f g}$$

$U_1 = U_2; z_1 - z_2 = -H = -0.25 \text{ m.}$
Hence, \( p_1 - p_2 = 2730 \text{ Pa} \).

Difference in pressure between the base and the top of the bed = 2730 Pa.

(b) Calculate bed height and mean bed voidage at a flow rate of 7 cm\(^3\)/s.

Apply Richardson-Zaki equation (Text-Equation 7.21), \( U = U_T \varepsilon^n \)

Superficial liquid velocity, \( U = \frac{\text{volume flow rate}}{\text{cross sectional area}} = \frac{7 \times 10^{-6}}{\frac{\pi 0.3^2}{4}} = 9.9 \times 10^{-5} \text{ m/s} \)

To determine the single particle terminal velocity, \( U_T \), assume Stokes Law (Text-Equation 2.13)

\[
U_T = \frac{x^2 g (\rho_p - \rho_f)}{18 \mu}
\]

with \( x = 70 \mu m, \rho_p = 1200 \text{ kg/m}^3, \rho_f = 1000 \text{ kg/m}^3 \) and \( \mu = 0.001 \text{ Pa.s} \),

\( U_T = 5.34 \times 10^{-4} \text{ m/s} \).

To determine exponent \( n \), calculate single particle Reynolds number \( Re_p \) at \( U = U_T \).

\[
Re_p = \frac{U_T \rho X}{\mu} = \frac{(5.34 \times 10^{-4}) \times 1000 \times (70 \times 10^{-6})}{0.001} = 0.037, \text{ which is less than 0.3. Hence, } n = 4.65 \text{ (Text-Equation 7.22)}
\]

Hence, applying the Richardson-Zaki equation, \( 9.9 \times 10^{-5} = (5.34 \times 10^{-4}) \varepsilon^{4.65} \)

gives, \( \varepsilon = 0.696 \)

From Equation 7.24, mass of solids in the bed, \( M = (1 - \varepsilon) \rho_p AH \)

Hence, bed height, \( H = \frac{12}{1200 \times (1 - 0.696) \times \left( \frac{\pi 0.3^2}{4} \right)} = 0.465 \text{ m.} \)
EXERCISE 7.6:
A packed bed of solids of density 2000 kg/m³ occupies a depth of 0.6 m in a cylindrical vessel of inside diameter 0.1 m. The mass of solids in the bed is 5 kg and

the surface-volume mean diameter of the particles is 300 μm. Water (density 1000 kg/m³ and viscosity 0.001 Pa.s) flows upwards through the bed.

a) What is the voidage of the packed bed?
b) Use a force balance over the bed to determine the bed pressure drop when fluidized.
c) Hence, assuming laminar flow and that the voidage at incipient fluidization is the same as the packed bed voidage, determine the minimum fluidization velocity. Verify the assumption of laminar flow.

SOLUTION TO EXERCISE 7.6:
(a) Cross-sectional area of bed, \( A = \frac{\pi 0.1^2}{4} - 7.85 \times 10^{-3} \) m²

From Equation 7.24, calculate bed voidage:

mass of solids in the bed, \( M = (1 - \varepsilon) \rho_p A H \)

Hence, voidage, \( \varepsilon = 1 - \frac{5}{2000 \times 7.85 \times 10^{-3} \times 0.6} = 0.4692 \)

(b) Force balance on bed. Apply Text-Equation 7.2:

\( \Delta p = H(1 - \varepsilon)(\rho_p - \rho_f)g \)

\( (-\Delta p) = 0.6 \times (1 - 0.4692) \times (2000 - 1000) \times 9.81 = 3124 \) Pa.

Pressure drop across the bed when fluidized = 3124 Pa.

(c) Assuming laminar flow through the bed, we apply only the laminar component of the Ergun equation.

Hence, \( \frac{(-\Delta p)}{H} = 150 \frac{(1 - \varepsilon)^2 \mu U}{\varepsilon x_{SV}} \)

With \( (-\Delta p) = 3124 \) Pa; \( \mu = 0.001 \) Pa.s; \( \rho_f = 1000 \) kg/m³; \( H = 0.6 \) m;
\( x_{SV} = 300 \times 10^{-6} \) m, and assuming the voidage of the bed at minimum fluidization is equal to the packed bed voidage, \( \varepsilon = 0.4692 \) then:

\( U - U_{mf} = 1.145 \times 10^{-3} \) m/s

Check Reynolds number for use of laminar flow in packed bed.

\( Re^* = \frac{U_{mf} \rho \mu x_{SV}}{\mu (1 - \varepsilon)} = 0.647 \), which is less than 10, the nominal upper limit for laminar flow. Hence the assumption of laminar flow is justified and \( U_{mf} = 1.145 \) mm/s.