Week # 10
MR Chapter 8

• Tutorial #10
• MR #8.1, 8.2.

• To be discussed on March 31, 2021.
• By either volunteer or class list.

MARTIN RHODES (2008)
Introduction to Particle Technology, 2nd Edition.
Publisher John Wiley & Son, Chichester, West Sussex, England.
Pneumatic Transport

- Gases have been used successfully in industry to transport a wide range of particulate solids.
- Most pneumatic transport was done in dilute suspension using large volumes of air at high velocity.
- There has been increasing interest in the so-called ‘dense phase’ mode of transport in which the solid particles are not fully suspended.
- Advantage of dense phase transport is low air requirements.
- Generally means a lower energy requirement.
- Resulting low solids velocities mean that product degradation by attrition and pipeline erosion are not major problems.
Dilute Phase and Dense Phase Transport

- Pneumatic transport of particulate solids is classified into: dilute (or lean) phase flow and dense phase flow.
- Dilute phase flow is characterized by high gas velocities (> 20 m/s), low solids concentrations (< 1% by volume) and low pressure drops per unit length of transport line (< 5 mbar/m).
- Dilute phase pneumatic transport is limited to short route, continuous transport of solids at rates of less than 10 t/h.
- Only system capable of operation under negative pressure.
- Solid particles behave as individuals, fully suspended in the gas.
- Fluid-particle forces dominate.
• Dense phase flow is characterized by low gas velocities (1 – 5 m/s), high solids concentrations (> 30% by volume) and high pressure drops per unit length of pipe (> 20 mbar/m)
• Particles are not fully suspended and there is much interaction between particles
• Boundary between dilute and dense phase flow is not clear cut and there are no universally accepted definitions
• Here, ‘choking’ and ‘saltation’ velocities will be used to mark the boundaries between dilute and dense phase transport in vertical and horizontal pipelines respectively
Choking Velocity in Vertical Transport

• Pressure drop across a length of transport line has, in general, six components
• Pressure drop due to gas acceleration, particle acceleration, gas-to-pipe friction, solid-to-pipe friction, static head of solids, static head of gas
• Line AB represents the frictional pressure loss due to gas only in a vertical transport line
• Curve CDE is for solids flux of $G_1$
• Curve FG is for a higher feed rate $G_2$
• At point C, gas velocity is high, concentration is low and frictional resistance between gas and pipe wall predominates
• As gas velocity is decreased, frictional resistance decreases but, since concentration of the suspension increases, static head required to support these solids increases
• If gas velocity is decreased below point D then increase in static head outweighs decrease in frictional resistance and $\Delta p/\Delta L$ rises again
• In region DE, the decreasing velocity causes a rapid increase in solids concentration
• A point is reached when the gas can no longer entrain all the solids
• At this point, a flowing slugging fluidized bed is formed in the transport line
• The phenomenon is known as ‘choking’ and usually shows large pressure fluctuations
• The choking velocity, $U_{CH}$, is the lowest velocity at which this dilute phase transport line can be operated at solids feed rate $G_1$
• At the higher solids feed rate, $G_2$, the choking velocity is higher
• Choking velocity marks the boundary between dilute and dense phase vertical pneumatic transport
• Choking can be reached by decreasing gas velocity at constant solids flow rate, or by increasing solids flow rate at constant gas velocity
• It is not possible to theoretically predict the conditions for choking to occur
• Correlations for predicting choking velocities are available

\[ \frac{U_{CH}}{\varepsilon_{CH}} - U_T = \frac{G}{\rho_p(1 - \varepsilon_{CH})} \]

\[ \rho_f^{0.77} = \frac{2250D(\varepsilon_{CH}^{4.7} - 1)}{(U_{CH}/\varepsilon_{CH} - U_T)^2} \]

• Where \( \varepsilon_{CH} \) is the voidage in the pipe at choking velocity \( U_{CH} \), \( \rho_p \) is the particle density, \( \rho_f \) is the gas density, \( G \) is the mass flux of solids \((=M_p/A)\) and \( U_T \) is the free fall or terminal velocity of a single particle in the gas
Saltation Velocity in Horizontal Transport

- General relationship between gas velocity and pressure gradient for horizontal transport line is in many ways similar to that for vertical transport.

- Line AB represents the curve obtained for gas only in the line.

- CDEF for a solids flux $G_1$ and curve GH for a higher solids feed rate $G_2$. 

![Graph showing pressure gradient and superficial gas velocity relationship](image)
• At point C, gas velocity is sufficiently high to carry all solids in very dilute suspension
• Solid particles are prevented from settling to the walls of the pipe by turbulent eddies generated in the flowing gas
• If gas velocity is reduced while solids feed rate is kept constant, frictional resistance and $\Delta p/\Delta L$ decrease
• Solids move more slowly and solids concentration increases
• At point D, gas velocity is insufficient to maintain the solids in suspension and solids begin to settle out in the bottom of the pipe
• Gas velocity at which this occurs is termed the ‘saltation velocity’
• Further decrease in gas velocity results in rapid ‘salting out’ of solids and rapid increase in $\Delta p/\Delta L$ as the area available for flow of gas is restricted by settled solids
• In the region E and F some solids may move in dense phase flow along the bottom of the pipe while others travel in dilute phase flow in the gas in the upper part of the pipe.

• Saltation velocity marks the boundary between dilute and dense phase flow in horizontal pneumatic transport.

• As with vertical pneumatic transport, it is not possible to theoretically predict the conditions under which saltation will occur.

• Correlations for predicting saltation velocity are available.

\[
\frac{M_p}{\rho_f U_{salt}A} = \left(\frac{1}{10(1440x+1.96)}\right)\left(\frac{U_{salt}}{\sqrt{gD}}\right)^{(1100x+2.5)}
\]

where \( \frac{M_p}{\rho_f U_{salt}A} \) is the solids loading (mass flow rate of solids) / (mass flow rate of gas).

\( \frac{U_{salt}}{\sqrt{gD}} \) is the Froude number at saltation.

• \( U_{salt} \) is the superficial gas velocity at saltation when the mass flow rate of solids is \( M_p \), the pipe diameter is \( D \) and the particle size is \( x \).
Fundamentals

• ‘Superficial velocity’ for gas and solids (particles) are defined as:

  \[
  \begin{align*}
  \text{superficial gas velocity, } U_{fs} &= \frac{Q_f}{A} \\
  \text{superficial solids velocity, } U_{ps} &= \frac{Q_p}{A}
  \end{align*}
  \]

  Where subscript ‘s’ denotes superficial and subscripts ‘f’ and ‘p’ refer to fluid and particles respectively.

• Fraction of pipe cross-sectional area available for flow of gas is usually assumed to be equal to the volume fraction occupied by gas (the voidage or void fraction \( \varepsilon \)).

• Fraction of pipe area available for flow of solids is therefore \((1 - \varepsilon)\).

• Actual gas velocity

  \[
  U_f = \frac{Q_f}{A\varepsilon}
  \]

• Actual particle velocity

  \[
  U_p = \frac{Q_p}{A(1 - \varepsilon)}
  \]

• Superficial velocities are related to actual velocities by the equations:

  \[
  \begin{align*}
  U_f &= \frac{U_{fs}}{\varepsilon} \\
  U_p &= \frac{U_{ps}}{1 - \varepsilon}
  \end{align*}
  \]

• Relative velocity between particle and fluid \( U_{rel} \):

• This velocity is often also referred to as the ‘slip velocity’ \( U_{slip} \).
Consider a length of transport pipe into which are fed particles and gas at mass flow rates of $M_p$ and $M_f$ respectively.

Continuity equations for particles and gas are:

$$M_p = AU_p(1 - \varepsilon)\rho_p$$
$$M_f = AU_f\varepsilon\rho_f$$

Combining these continuity equations gives an expression for ratio of mass flow rates, known as the solids loading:

$$\text{Solids loading, } \frac{M_p}{M_f} = \frac{U_p(1 - \varepsilon)\rho_p}{U_f\varepsilon\rho_f}$$

This shows that the average voidage $\varepsilon$, at a particular position along the length of the pipe, is a function of the solids loading and the magnitudes of the gas and solids velocities for given gas and particle density.
• To obtain an expression for total pressure drop along a section of transport line, we will write down the momentum equation for a section of pipe.

• Consider a section of pipe of cross-sectional area $A$ and length $\delta L$ inclined to the horizontal at an angle $\theta$ and carrying a suspension of voidage $\varepsilon$. 
• Momentum balance equation is:

\[
\left( \text{net force acting on pipe contents} \right) = \left( \text{rate of increase in momentum of contents} \right)
\]

\[
\left( \begin{array}{c}
\text{(pressure)} \\
\text{force}
\end{array} \right) - \left( \begin{array}{c}
\text{(gas-wall friction force)} \\
\text{friction force}
\end{array} \right) - \left( \begin{array}{c}
\text{(solids-wall friction force)} \\
\text{friction force}
\end{array} \right) - \left( \begin{array}{c}
\text{(gravitational force)} \\
\text{rate of increase in momentum of the gas}
\end{array} \right) = \left( \begin{array}{c}
\text{(rate of increase in momentum of the solids)}
\end{array} \right)
\]

or

\[
-A \delta p - F_{fw} \delta L - F_{pw} \delta U_t = [A(1 - \varepsilon) \rho_p \delta L] g \sin \theta - (A \varepsilon \rho_t \delta L) g \sin \theta
\]

\[
= \rho_t A \varepsilon U_t \delta U_t + \rho_p A(1 - \varepsilon) U_p \delta U_p
\]

(8.14)

• Where \( F_{fw} \) and \( F_{pw} \) are the gas-to-wall friction force and solids-to-wall friction force per unit volume of pipe respectively.
Rearranging and integrating assuming constant gas density and voidage:

\[ p_1 - p_2 = \frac{1}{2} \varepsilon \rho_f U_i^2 + \frac{1}{2} (1 - \varepsilon) \rho_p U_p^2 + F_{f_w} L + F_{p_w} L + \rho_p L (1 - \varepsilon) g \sin \theta + \rho_i L \varepsilon g \sin \theta \]

Total pressure drop along a straight length of pipe carrying solids in dilute phase transport is made up of a number of terms:

- (1) pressure drop due to gas acceleration
- (2) pressure drop due to particle acceleration
- (3) pressure drop due to gas-to-wall friction
- (4) pressure drop related to solids-to-wall friction
- (5) pressure drop due to the static head of solids
- (6) pressure drop due to the static head of gas
Design for Dilute Phase Transport

• In both horizontal and vertical dilute phase transport, it is desirable to operate at the lowest possible velocity in order to minimize frictional pressure loss, reduce attrition and reduce running loss.

• For a particular pipe size and solids flow rate, saltation velocity is always higher than choking velocity.

• Therefore, in a transport system comprising both vertical and horizontal lines, gas velocity must be selected to avoid saltation.

• In this way choking will also be avoided.

• $U_{\text{salt}}$ and $U_{\text{CH}}$ cannot be predicted with confidence and conservative design is necessary.

• Bearing in mind the uncertainty in correlations for predicting choking and saltation velocities, safety margins of 50% and greater are recommended when selecting the operating gas velocity.
• In dilute transport, gas-to-wall friction is often assumed independent of the presence of solids and so friction factor for the gas may be used

• For vertical transport

\[ F_{pw}L = 0.057GL \sqrt{\frac{8}{D}} \]

• For horizontal transport

\[ F_{pw}L = \frac{2f_p(1 - \varepsilon) \rho_p U_p^2 L}{D} \]

\[ F_{pw}L = \frac{2f_p G U_p L}{D} \]

• Where

\[ U_p = U(1 - 0.0638x^{0.3} \rho_p^{0.5}) \]

\[ f_p = \frac{3}{8} \rho_f D x C_D \left( \frac{U_t - U_p}{U_p} \right)^2 \]

• Where \( C_D \) is the drag coefficient between particle and gas
• Above analysis assumes that particles lose momentum by collision with pipe walls
• Pressure loss due to solids-wall friction is the gas pressure loss as a result of re-accelerating the solids
• Drag force on a single particle is given by:

\[ F_D = \frac{\pi x^2}{4} \rho_t C_D \frac{(U_t - U_p)^2}{2} \]

• If the void fraction is \( \varepsilon \), then the number of particles per unit volume of pipe \( N_v \) is

\[ N_v = \frac{(1-\varepsilon)}{\pi x^3/6} \]

• Therefore the force exerted by the gas on the particles in unit volume of pipe \( F_v \) is

\[ F_v = F_D \frac{(1-\varepsilon)}{\pi x^3/6} \]

• This is equal to the solids-wall friction force per unit volume of pipe, \( F_{pw} \)

\[ F_{pw} L = \frac{3}{4} \rho_t C_D \frac{L}{x} (1-\varepsilon)(U_t - U_p)^2 \]
• Bends complicate the design of pneumatic dilute phase transport systems
• When designing a transport system it is best to use as few bends as possible
• Bends increase the pressure drop in a line and also are the points of most serious erosion and particle attrition
• Solids normally in suspension in straight, horizontal or vertical pipes tend to salt out at bends due to centrifugal force encountered while travelling around the bend
• Because of this operation, the particles slow down and are then re-entrained and re-accelerated after they pass through the bend, resulting in the higher pressure drops associated with bends
• There is greater tendency for particles to salt out in a horizontal pipe which is preceded by a downflowing vertical to horizontal bend than in any other configuration
• It is possible for solids to remain on the bottom of the pipe for very long distances following the bend before they redisperse
• Blinded tees can be used in place of elbows in pneumatic transport systems
• A cushion of stagnant particles collects in the blinded or unused branch of the tee
• The conveyed particles then impinge upon the stagnant particles in the tee rather than on the metal surface
• Service life of the blinded tee configuration is far better than any other configurations tested
• Service life 15 times greater than that of radius bends or elbows

In industrial practice, bend pressure drop is often approximated by assuming that it is equivalent to approximately 7.5 m of vertical section pressure drop
Dense Phase Transport

- Dense phase transport is described as the condition in which solids are conveyed such that they are not entirely suspended in the gas.
- Transition point between dilute and dense phase transport is saltation for horizontal transport and choking for vertical transport.
- However, even within the dense phase regime a number of different flow patterns occur in both horizontal and vertical transport.
- Continuous dense phase flow pattern in which the solids occupy the entire pipe is extrusion.
- Transport in this form requires very high gas pressures and is limited to short straight pipe lengths and granular materials which have a high permeability.
• Discontinuous dense phase flow can be divided into:
  • ‘Discrete plug flow’ in which discrete plugs of solids occupy the full pipe cross-section
  • ‘Dune flow’ in which a layer of solids settled at the bottom of the pipe move along in the form of rolling dunes
  • A hybrid of discrete plug flow and dune flow in which rolling dunes completely fill the pipe cross-section but in which there are not discrete plugs (also known as ‘plug flow’)

![Diagram showing various types of dense phase flow and their relationship to pressure gradient and gas velocity.](image-url)
• Saltating flow is encountered at gas velocities just below the saltation velocity
• Particles are conveyed in suspension above a layer of settled solids
• Particles may be deposited and re-entrained from this layer
• As the gas velocity is decreased the thickness of the layer of settled solids increases and eventually form dune flow
• Main advantages of dense phase transport arise from the low gas requirements and low solids velocities
• Low gas volume requirements generally mean low energy requirements per kg of product conveyed
• Also mean that smaller pipelines and recovery and solids-gas separation required
• Low solids velocities means that abrasive and friable materials may be conveyed without major pipeline erosion or product degradation
• Design of commercial dense phase systems is largely empirically based
8.4 WORKED EXAMPLES

WORKED EXAMPLE 8.1

Design a positive pressure dilute-phase pneumatic transport system to transport 900 kg/h of sand of particle density 2500 kg/m³ and mean particle size 100 μm between two points in a plant separated by 10 m vertical distance and 30 m horizontal distance using ambient air. Assume that six 90° bends are required and that the allowable pressure loss is 0.55 bar.

Solution

In this case, to design the system means to determine the pipe size and air flow rate which would give a total system pressure loss near to the allowable pressure loss.

The design procedure requires trial and error calculations. Pipes are available in fixed sizes and so the procedure adopted here is to select a pipe size and determine the saltation velocity from Equation (8.1). The system pressure loss is then calculated at a superficial gas velocity equal to 1.5 times the saltation velocity [this gives a reasonable safety margin bearing in mind the accuracy of the correlation in Equation (8.1)]. The calculated system pressure loss is then compared with the allowable pressure loss. The pipe size selected may then be altered and the above procedure repeated until the calculated pressure loss matches that allowed.
Step 1. Selection of pipe size

Select 78 mm inside diameter pipe.

Step 2. Determine gas velocity

Use the Rizk correlation of Equation (8.3) to estimate the saltation velocity, $U_{salt}$. Equation (8.3) rearranged becomes

$$U_{salt} = \left( \frac{4M_p 10^x g^{\beta/2} D^{(\beta/2)-2}}{\pi \rho_i} \right)^{1/(\beta+1)}$$

where $x = 1440x + 1.96$ and $\beta = 1100x + 2.5$.

In the present case $x = 2.104$, $\beta = 2.61$ and $U_{salt} = 9.88$.

Therefore, superficial gas velocity, $U = 1.5 \times 9.88 \text{ m/s} = 14.82 \text{ m/s}$.
Step 3. Pressure loss calculations

(a) Horizontal sections. Starting with Equation (8.15) an expression for the total pressure loss in the horizontal sections of the transport line may be generated. We will assume that all the initial acceleration of the solids and the gas take place in the horizontal sections and so terms (1) and (2) are required. For term (3) the Fanning friction equation is used assuming that the pressure loss due to gas-to-wall friction is independent of the presence of solids. For term (4) we employ the Hinkle correlation [Equation (8.17)]. Terms (5) and (6) become zero as \( \theta = 0 \) for horizontal pipes. Thus, the pressure loss, \( \Delta p_H \), in the horizontal sections of the transport line is given by:

\[
\Delta p_H = \frac{\rho_f e_H U_{H}^2}{2} + \frac{\rho_p (1 - e_H) U_{PH}^2}{2} + \frac{2f_g \rho_f U^2 L_H}{D} + \frac{2f_p \rho_p (1 - e_H) U_{PH}^2 L_H}{D}
\]

where the subscript H refers to the values specific to the horizontal sections.

To use this equation we need to know \( e_H \), \( U_{H} \), and \( U_{PH} \). Hinkle’s correlation gives us \( U_{PH} \):

\[
U_{PH} = U(1 - 0.0638 x^{0.3} \rho_p^{0.5}) = 11.84 \text{ m/s}
\]
From continuity, \( G = \rho_p (1 - \varepsilon_H) U_{pH} \)

thus \( \varepsilon_H = 1 - \frac{G}{\rho_p U_{pH}} = 0.9982 \)

and \( U_{tH} = \frac{U}{\varepsilon_H} = \frac{14.82}{0.9982} = 14.85 \text{ m/s} \)

Friction factor \( f_p \) is found from Equation (8.19) with \( C_D \) estimated at the relative velocity \( (U_t - U_p) \), using the approximate correlations given below [or by using an appropriate \( C_D \) versus \( Re \) chart (see Chapter 2)]:

\[
\begin{align*}
Re_p < 1 & \quad C_D = 24/Re_p \\
1 < Re_p < 500 & \quad C_D = 18.5Re_p^{-0.6} \\
500 < Re_p < 2 \times 10^5 & \quad C_D = 0.44
\end{align*}
\]

Thus, for flow in the horizontal sections,

\[
Re_p = \frac{\rho_t (U_{tH} - U_{pH}) x}{\mu}
\]

for ambient air \( \rho_t = 1.2 \text{ kg/m}^3 \) and \( \mu = 18.4 \times 10^{-6} \text{ Pa s} \), giving

\[
Re_p = 19.63
\]

and so, using the approximate correlations above,

\[
C_D = 18.5Re_p^{-0.6} = 3.1
\]

Substituting \( C_D = 3.1 \) in Equation (8.19) we have:

\[
f_p = \frac{3}{8} \times \frac{1.2}{2500} \times 3.1 \times \frac{0.078}{100 \times 10^{-6}} \left( \frac{14.85 - 11.84}{11.84} \right)^2
\]

The gas friction factor is taken as \( f_g = 0.005 \). This gives \( \Delta p_H = 14.864 \text{ Pa} \).
(b) Vertical sections. Starting again with Equation (8.15), the general pressure loss equation, an expression for the total pressure loss in the vertical section may be derived. Since the initial acceleration of solids and gas was assumed to take place in the horizontal sections, terms (1) and (2) become zero. The Fanning friction equation is used to estimate the pressure loss due to gas-to-wall friction [term (3)] assuming solids have negligible effect on this pressure loss. For term (4) the modified Konno and Saito correlation [Equation (8.16)] is used. For vertical transport $\theta$ becomes equal to 90° in terms (5) and (6).

Thus, the pressure loss, $\Delta p_v$, in the vertical sections of the transport line is given by:

$$\Delta p_v = \frac{2f_{g,p} U_r^2 L_v}{D} + 0.057 G L_v \sqrt{\frac{g}{D}} + \rho_p (1 - \varepsilon_v) g L_v + \rho_f \varepsilon_v g L_v$$

where subscript $v$ refers to values specific to the vertical sections.

To use this equation we need to calculate the voidage of the suspension in the vertical pipe line $\varepsilon_v$.

Assuming particles behave as individuals, then slip velocity is equal to single particle terminal velocity, $U_T$ (also noting that the superficial gas velocity in both horizontal and vertical sections is the same and equal to $U$, i.e.

$$U_{pv} = \frac{U}{\varepsilon_v} - U_T$$

continuity gives particle mass flux, $G = \rho_p (1 - \varepsilon_v) U_{pv}$.

Combining these equations gives a quadratic in $\varepsilon_v$ which has only one possible root.

$$\varepsilon_v^2 U_T - \left( U_T + U + \frac{G}{\rho_p} \right) \varepsilon_v + U = 0$$

The single particle terminal velocity, $U_T$ may be estimated as shown in Chapter 2, giving $U_T = 0.52 \text{ m/s}$ assuming the particles are spherical.

And so, solving the quadratic equation, $\varepsilon_v = 0.9985$ and thus $\Delta p_v = 1148 \text{ Pa.}$
(c) Bends. The pressure loss across each 90° bend is taken to be equivalent to that across 7.5 m of vertical pipe.

Pressure loss per metre of vertical pipe = \( \frac{\Delta p_v}{L_v} = 114.8 \text{ Pa/m} \)

Therefore, pressure loss across six 90° bends

\[
= 6 \times 7.5 \times 114.8 \text{ Pa}
= 5166 \text{ Pa}
\]

And so,

\[
\text{(total pressure loss)} = \text{(loss across vertical sections)} + \text{(loss across horizontal sections)} + \text{(loss across bends)}
\]

\[
= 11.48 + 14864 + 5166 \text{ Pa}
= 0.212 \text{ bar}
\]
Step 4. Compare calculated and allowable pressure losses

The allowable system pressure loss is 0.55 bar and so we may select a smaller pipe size and repeat the above calculation procedure. The table below gives the results for a range of pipe sizes.

<table>
<thead>
<tr>
<th>Pipe inside diameter (mm)</th>
<th>Total system pressure loss (bar)</th>
</tr>
</thead>
<tbody>
<tr>
<td>78</td>
<td>0.212</td>
</tr>
<tr>
<td>63</td>
<td>0.322</td>
</tr>
<tr>
<td>50</td>
<td>0.512</td>
</tr>
<tr>
<td>40</td>
<td>0.809</td>
</tr>
</tbody>
</table>

In this case we would select 50 mm pipe which gives a total system pressure loss of 0.512 bar. (An economic option could be found if capital and running cost were incorporated.) The design details for this selection are given below:

pipe size = 50 mm inside diameter
air flow rate = 0.0317 m³/s
air superficial velocity = 16.15 m/s
saltation velocity = 10.77 m/s
solids loading = 6.57 kg solid/kg air
total system pressure loss = 0.512 bar