Week # 12
MR Chapters 11 & 12

• Tutorial #12
• MR #11.2, 12.1.

MARTIN RHODES (2008)
Introduction to Particle Technology, 2nd Edition.
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Mixing and Segregation

- Achieving good mixing of particulate solids of different size and density is important in many process industries.
- A perfect mixture of two types of particles is one in which a group of particles taken from any position in the mixture will contain the same proportions of each particle as the proportions present in the whole mixture.
- A random mixture is a mixture in which the probability of finding a particle of any component is the same at all locations and equal to the proportion of that component in the mixture as a whole.
Segregation

- In many systems, particles to be mixed have different properties and tend to exhibit segregation.
- Particles with the same physical property collect together in one part of the mixture and random mixture is not a natural state.
- Even if particles are originally mixed by some means, they will tend to unmix on handling (moving, pouring, conveying, processing).
- Differences in size, density and shape of constituent particles of a mixture may give rise to segregation.
- Difference in particle size is most important, density difference is comparatively unimportant except in gas fluidization.
- Demixing or segregation can give rise to variations in bulk density of powder going to packaging.
- Chemical composition of the product may be off specification (e.g. in blending of constituents for detergents or drugs).
• Four mechanisms of segregation according to size may be identified:

• (1) Trajectory segregation: if a small particle of diameter x and density \( \rho_p \), whose drag is governed by Stokes’ law is projected horizontally with a velocity \( U \) into a fluid of viscosity \( \mu \) and density \( \rho_f \), the limiting distance that it can travel horizontally is \( U\rho_px^2/36\mu \)

• A particle of diameter 2x would travel four times as far before coming to rest

• This mechanism can cause segregation where particles are caused to move through air or when powders fall from the end of a conveyor belt

From Chapter 2, the retarding force on the particle = \( C_D \frac{1}{2} \rho_i U^2 \left( \frac{\pi x^2}{4} \right) \)

Deceleration of the particle = \( \frac{\text{retarding force}}{\text{mass of particle}} \)

In Stokes’ law region, \( C_D = 24/Re_p \)

Hence, deceleration = \( \frac{18U\mu}{\rho_p x^2} \)

From the equation of motion a particle with an initial velocity \( U \) and constant deceleration \( 18U\mu/\rho_p x^2 \) will travel a distance \( U\rho_px^2/18\mu \) before coming to rest.

Trajectory segregation
(2) Percolation of fine particles: if a mass of particles is disturbed in such a way that individual particles move, a rearrangement in the packing of the particles occurs.

The gaps created allow particles from above to fall and particles in some other place to move upwards.

If the powder is composed of particles of different size, it will be easier for small particles to fall down and so there will be a tendency for small particles to move downwards leading to segregation.

Even a very small difference in particle size can give rise to significant segregation.

Segregation by percolation of fine particles can occur whenever the mixture is disturbed causing rearrangement of particles.

This can happen during stirring, shaking, vibration or when pouring particles into a heap.

Segregation by percolation occurs in charging and discharging storage hoppers.

As particles are fed into a hopper they generally pour into a heap resulting in segregation if there is a size distribution and the powder is free-flowing.
• (3) Rise of coarse particles on vibration: if a mixture of particles of different size is vibrated the larger particles move upwards
• This is the so-called ‘Brazil-nut effect’ and has received much attention in recent years
• The rise of the larger or denser ‘intruder’ within the bed of smaller particles has been explained in terms of creation and filling of voids beneath the intruder
• (4) Elutriation segregation: when a powder containing an appreciable proportion of particles under 50 μm is charged into a storage vessel or hopper, air is displaced upwards

• The upward velocity of air may exceed the terminal freefall velocity of some of the finer particles, which may then remain in suspension after the larger particles have settled

• Thus a pocket of fine particles is generated in the hopper each time solids are charged
Reduction of Segregation

• Segregation occurs primarily as a result of size difference
• Difficulty of mixing two components can be reduced by making the size of the components as similar as possible and by reducing the absolute size of both components
• Segregation is generally not a serious problem when all particles are less than 30 μm (for particle densities in the range 2000 – 3000 kg/m³)
• In such fine powders, interparticle forces generated by electrostatic charging, van der Waals forces and forces due to moisture are large compared with gravitational and inertial forces
• This causes particles to stick together preventing segregation as particles are not free to move relative to one another
• The mobility of particles in free-flowing powders can be reduced by addition of small quantities of liquid
• The reduction in mobility reduces segregation and permits better mixing
Three mechanisms of powder mixing:

1. Shear mixing: shear stresses give rise to slip zones and mixing takes place by interchange of particles between layers within the zone.

2. Diffusive mixing: occurs when particles roll down a sloping surface.

3. Convective mixing: deliberate bulk movement of packets of powder around the powder mass.

In free-flowing powders both diffusive mixing and shear mixing give rise to size segregation.

For such powders, convective mixing is the major mechanism promoting mixing.

Types of mixers: tumbling mixers, convective mixers, fluidized bed mixers, high shear mixers.
Sampling

- To determine the quality of a mixture, it is generally necessary to take samples.
- Sampling of mixtures and analysis of mixture quality require application of statistical methods.
- Mean composition: the true composition of a mixture $\mu$ is often not known but an estimate may be found by sampling.
- Statistics relevant to random binary mixtures are as follows:
- For $N$ samples of composition $y_1$ to $y_N$ in one component, the estimate of the mixture composition is given by:

\[ \bar{y} = \frac{1}{N} \sum_{i=1}^{N} y_i \]
• Standard deviation and variance: the true standard deviation, \( \sigma \), and the true variance, \( \sigma^2 \), of the composition of the mixture are quantitative measures of the quality of the mixture.

• The true variance is usually not known but an estimate \( S^2 \) is defined as:

\[
S^2 = \frac{\sum_{i=1}^{N} (y_i - \mu)^2}{N} \quad \text{if the true composition } \mu \text{ is known}
\]

\[
S^2 = \frac{\sum_{i=1}^{N} (y_i - \bar{y})^2}{N-1} \quad \text{if the true composition } \mu \text{ is unknown}
\]

• The standard deviation is equal to the square root of variance.
• Theoretical limits of variance: for a two-component system the theoretical upper and lower limits of mixture variance are:

(a) upper limit (completely segregated) \( \sigma_0^2 = p(1-p) \)
(b) lower limit (randomly mixed) \( \sigma_R^2 = \frac{p(1-p)}{n} \)

• Where \( p \) and \( 1-p \) are the proportions of the two components determined from samples and \( n \) is the number of particles in each sample

• Mixing indices: a measure of the degree of mixing is the Lacey mixing index

\[
\text{Lacey mixing index} = \frac{\sigma_0^2 - \sigma_R^2}{\sigma_0^2 - \sigma_R^2}
\]

• In practical terms the Lacey mixing index is the ratio of ‘mixing achieved’ to ‘mixing possible’

• A Lacey mixing index of zero would represent complete segregation and a value of unit would represent a completely random mixture

• Practical values of this mixing index are found to lie in the range 0.75 to 1.0

• A further mixing index is defined as:

\[
\text{Poole et al. mixing index} = \frac{\sigma}{\sigma_R}
\]

• This index gives better discrimination for practical mixtures and approaches unity for completely random mixtures
WORKED EXAMPLE 11.1 (AFTER WILLIAMS, 1990)

A random mixture consists of two components A and B in proportions 60 and 40% by mass, respectively. The particles are spherical and A and B have particle densities 500 and 700 kg/m³, respectively. The cumulative undersize mass distributions of the two components are shown in Table 11W.1.

If samples of 1 g are withdrawn from the mixture, what is the expected value for the standard deviation of the composition of the samples?

<table>
<thead>
<tr>
<th>Size $x$ (µm)</th>
<th>2057</th>
<th>1676</th>
<th>1405</th>
<th>1204</th>
<th>1003</th>
<th>853</th>
<th>699</th>
<th>599</th>
<th>500</th>
<th>422</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_A(x)$</td>
<td>1.00</td>
<td>0.80</td>
<td>0.50</td>
<td>0.32</td>
<td>0.19</td>
<td>0.12</td>
<td>0.07</td>
<td>0.04</td>
<td>0.02</td>
<td>0</td>
</tr>
<tr>
<td>$F_B(x)$</td>
<td>1.00</td>
<td>0.88</td>
<td>0.68</td>
<td>0.44</td>
<td>0.21</td>
<td>0.08</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
Solution

The first step is to estimate the number of particles per unit mass of A and B. This is done by converting the size distributions into differential frequency number distributions and using:

\[
\frac{\text{mass of particles}}{\text{in each size range}} = \frac{\text{number of particles}}{\text{in size range}} \times \frac{\text{mass of one particle}}{\rho_p \pi x^3} \frac{\rho_p \pi x^3}{6}
\]

where \(\rho_p\) is the particle density and \(x\) is the arithmetic mean of adjacent sieve sizes.

These calculations are summarized in Tables 11W1.2 and 11W1.3.

<table>
<thead>
<tr>
<th>Table 11W.1.2</th>
<th>A particles</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean size of range (x) ((\mu)m)</td>
<td>(d m)</td>
</tr>
<tr>
<td>----------------</td>
<td>---------</td>
</tr>
<tr>
<td>1866.5</td>
<td>0.20</td>
</tr>
<tr>
<td>1540.5</td>
<td>0.30</td>
</tr>
<tr>
<td>1304.5</td>
<td>0.18</td>
</tr>
<tr>
<td>1103.5</td>
<td>0.13</td>
</tr>
<tr>
<td>928</td>
<td>0.07</td>
</tr>
<tr>
<td>776</td>
<td>0.05</td>
</tr>
<tr>
<td>649</td>
<td>0.03</td>
</tr>
<tr>
<td>54.5</td>
<td>0.02</td>
</tr>
<tr>
<td>461</td>
<td>0.02</td>
</tr>
<tr>
<td>Totals</td>
<td>1.00</td>
</tr>
<tr>
<td>Mean size of range ( x ) (( \mu \text{m} ))</td>
<td>( d , m )</td>
</tr>
<tr>
<td>-----------------</td>
<td>------</td>
</tr>
<tr>
<td>1866.5</td>
<td>0</td>
</tr>
<tr>
<td>1540.5</td>
<td>0</td>
</tr>
<tr>
<td>1304.5</td>
<td>0.12</td>
</tr>
<tr>
<td>1103.5</td>
<td>0.20</td>
</tr>
<tr>
<td>928</td>
<td>0.24</td>
</tr>
<tr>
<td>776</td>
<td>0.23</td>
</tr>
<tr>
<td>649</td>
<td>0.13</td>
</tr>
<tr>
<td>54.5</td>
<td>0.08</td>
</tr>
<tr>
<td>461</td>
<td>0</td>
</tr>
<tr>
<td>Totals</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Thus \( n_A = 3.51 \times 10^6 \) particles per kg

and

\( n_B = 5.33 \times 10^6 \) particles per kg

And in samples of 1 g (0.001 kg) we would expect a total number of particles:

\[
\begin{align*}
  n &= 0.001 \times (3.51 \times 10^6 \times 0.6 + 5.33 \times 10^6 \times 0.4) \\
  &= 4238 \text{ particles}
\end{align*}
\]

And so, from Equation (11.5) for a random mixture,

\[
\text{standard deviation}, \quad \sigma = \sqrt{\frac{0.6 \times 0.4}{4238}} = 0.0075
\]
WORKED EXAMPLE 11.3

During the mixing of a drug with an excipient the standard deviation of the compositions of 100 mg samples tends to a constant value of ±0.005. The size distributions of drug (D) and excipient (E) are given in Table 11W3.1.

Table 11W3.1 Size distributions of drug and excipient

<table>
<thead>
<tr>
<th>Size x (μm)</th>
<th>420</th>
<th>355</th>
<th>250</th>
<th>190</th>
<th>150</th>
<th>75</th>
<th>53</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_D(x)$</td>
<td>1.00</td>
<td>0.991</td>
<td>0.982</td>
<td>0.973</td>
<td>0.964</td>
<td>0.746</td>
<td>0.047</td>
<td>0</td>
</tr>
<tr>
<td>$F_E(x)$</td>
<td>1.00</td>
<td>1.00</td>
<td>0.977</td>
<td>0.967</td>
<td>0.946</td>
<td>0.654</td>
<td>0.284</td>
<td>0</td>
</tr>
</tbody>
</table>

The mean proportion by mass of drug is known to be 0.2. The densities of drug and excipient are 1100 and 900 kg/m³, respectively.

Determine whether the mixing is satisfactory (a) if the criterion is a random mixture and (b) if the criterion is an in-house specification that the composition of 95% of the samples should lie within ±15% of the mean.

Solution

The number of particles of drug (Table 11W3.2) and excipient (Table 11W3.3) in each sample is first calculated as shown in Worked Example 11.1.

Thus $n_D = 8.96 \times 10^9$ particles per kg
### Table 11W3.2 Number of drug particles in each kg of sample

<table>
<thead>
<tr>
<th>Mean size of range $x$ (μm)</th>
<th>$dm$</th>
<th>$dn$</th>
</tr>
</thead>
<tbody>
<tr>
<td>388</td>
<td>0.009</td>
<td>$2.67 \times 10^5$</td>
</tr>
<tr>
<td>303</td>
<td>0.009</td>
<td>$5.62 \times 10^5$</td>
</tr>
<tr>
<td>220</td>
<td>0.009</td>
<td>$1.47 \times 10^6$</td>
</tr>
<tr>
<td>170</td>
<td>0.009</td>
<td>$3.18 \times 10^6$</td>
</tr>
<tr>
<td>113</td>
<td>0.218</td>
<td>$2.62 \times 10^8$</td>
</tr>
<tr>
<td>64</td>
<td>0.700</td>
<td>$4.64 \times 10^9$</td>
</tr>
<tr>
<td>27</td>
<td>0.046</td>
<td>$4.06 \times 10^9$</td>
</tr>
<tr>
<td>20</td>
<td>0.00</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0.00</td>
<td>0</td>
</tr>
<tr>
<td><strong>Totals</strong></td>
<td>1.00</td>
<td>$8.96 \times 10^9$</td>
</tr>
</tbody>
</table>

### Table 11W3.3 Number of excipient particles in each kg of sample

<table>
<thead>
<tr>
<th>Mean size of range $x$ (μm)</th>
<th>$dm$</th>
<th>$dn$</th>
</tr>
</thead>
<tbody>
<tr>
<td>388</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>303</td>
<td>0.023</td>
<td>$1.75 \times 10^6$</td>
</tr>
<tr>
<td>220</td>
<td>0.010</td>
<td>$1.99 \times 10^6$</td>
</tr>
<tr>
<td>170</td>
<td>0.021</td>
<td>$9.07 \times 10^6$</td>
</tr>
<tr>
<td>113</td>
<td>0.292</td>
<td>$4.29 \times 10^8$</td>
</tr>
<tr>
<td>64</td>
<td>0.374</td>
<td>$3.03 \times 10^9$</td>
</tr>
<tr>
<td>27</td>
<td>0.28</td>
<td>$3.02 \times 10^9$</td>
</tr>
<tr>
<td>20</td>
<td>0.00</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td><strong>Totals</strong></td>
<td>1.00</td>
<td>$3.37 \times 10^{10}$</td>
</tr>
</tbody>
</table>
\[ n_E = 3.37 \times 10^{10} \text{ particles per kg} \]

And in samples of 1 g (0.001 kg) we would expect a total number of particles:

\[ n = 100 \times 10^{-6} \times (8.96 \times 10^9 \times 0.2 + 3.37 \times 10^{10} \times 0.8) \]
\[ = 2.88 \times 10^6 \text{ particles} \]

And so, from Equation (11.5) for a random mixture,

\[ \sigma_R = \sqrt{\frac{0.2 \times 0.8}{2.88 \times 10^6}} = 0.000235 \]

**Conclusion.** The actual standard deviation of the mixture is greater than that for a random mixture and so the criterion of random mixing is not achieved.

For a normal distribution the in-house criterion that 95% of samples should lie within \( \pm 15\% \) of the mean suggests that:

\[ 1.96\sigma = 0.15 \times 0.2 \]

(since for a normal distribution 95% of the values lie within \( \pm 1.96 \) standard deviations of the mean).

Hence, \( \sigma = 0.0153 \). So the in-house criterion is achieved.
Size Reduction

• Size reduction or comminution is an important step in the processing of many solid materials
• It may be used to create particles of a certain size and shape, to increase surface area available for chemical reaction
• Size reduction of solids is an energy intensive and highly inefficient process
• Design and scale-up of comminution processes is usually based on experience and testing
Energy Requirement and Product Size Distribution

- There are three well-known postulates predicting energy requirements for particle size reductions:
  - Rittinger (1867) proposed that the energy required for particle size reduction is directly proportional to the area of new surface created.
  - If initial and final particle sizes are $x_1$ and $x_2$ respectively, then assuming a volume shape factor $k_v$ independent of size,
    
    \[
    \text{volume of initial particle} = k_v x_1^3 \\
    \text{volume of final particle} = k_v x_2^3
    \]
  - If the surface shape factor $k_s$ is also independent of size, then for each original particle, the new surface created upon reduction is given by:
    \[
    \left(\frac{x_1^3}{x_2^3}\right)k_s x_2^2 - k_s x_1^2
    \]
  - Which simplifies to:
    \[
    k_s x_1^3 \left(\frac{1}{x_2} - \frac{1}{x_1}\right)
    \]
Therefore, new surface created per unit mass of original particles

$$= k_s x_1^3 \left( \frac{1}{x_2} - \frac{1}{x_1} \right) \times \left( \text{number of original particles per unit mass} \right)$$

$$= k_s x_1^3 \left( \frac{1}{x_2} - \frac{1}{x_1} \right) \times \left( \frac{1}{k_v x_1^3 \rho_p} \right)$$

$$= \frac{k_s}{k_v \rho_p} \left( \frac{1}{x_2} - \frac{1}{x_1} \right)$$

Where $\rho_p$ is the particle density

Hence assuming shape factors and density are constant, Rittinger’s postulate may be expressed as:

$$\text{breakage energy per unit mass of feed}, E = C_R \left( \frac{1}{x_2} - \frac{1}{x_1} \right)$$

Where $C_R$ is a constant

If this is the integral form, then in differential form, Rittinger’s postulate becomes

$$\frac{dE}{dx} = -C_R \frac{1}{x^2}$$
• On the basis of stress analysis theory for plastic deformation, Kick (1885) proposed that the energy required in any comminution process was directly proportional to the ratio of the volume of the feed particle to the product particle.

\[
\frac{v_1}{v_2} = \frac{x_1^3}{x_2^3}, \text{ determines the energy requirement (assuming shape factor is constant)}
\]

• Therefore, size ratio, \( \frac{x_1}{x_2} \) fixes the volume ratio, \( \frac{x_1^3}{x_2^3} \) which determines the energy requirement.

• And so, if \( \Delta x_1 \) is the change in particle size,

\[
\frac{x_2}{x_1} = 1 - \frac{\Delta x_1}{x_1}
\]

• Which fixes volume ratio, \( \frac{x_1^3}{x_2^3} \) and determines the energy requirement.
• So, $\Delta x_1/x_1$ determines the energy requirement for particle size reduction from $x_1$ to $x_1 - \Delta x_1$

\[ \Delta E = C_K \left( \frac{\Delta x}{x} \right) \]

• As $\Delta x_1 \to 0$,

\[ \frac{dE}{dx} = C_K \frac{1}{x} \]

• This is Kick’s law in differential form ($C_K$ is the Kick’s law constant)

• Integrating,

\[ E = C_K \ln \left( \frac{x_1}{x_2} \right) \]

• This proposal is unrealistic in most cases since it predicts that the same energy is required to reduce 10 $\mu$m particles to 1 $\mu$m particles as is required to reduce 1 m boulders to 10 cm block
• Bond (1952) suggested a more useful formula:

\[ E = C_B \left( \frac{1}{\sqrt{x_2}} - \frac{1}{\sqrt{x_1}} \right) \]

• However, Bond’s law is usually presented in the form shown below:

\[ E_B = W_1 \left( \frac{10}{\sqrt{X_2}} - \frac{10}{\sqrt{X_1}} \right) \]

• Where \( E_B \) is the energy required to reduce the top particle size of the material from \( x_1 \) to \( x_2 \) and \( W_1 \) is the Bond work index.

• The law is based on data which Bond obtained from industrial and laboratory scale processes involving many materials.

• Since top size is difficult to define, in practice \( X_1 \) to \( X_2 \) are taken to be the sieve size in micrometers through which 80% of the material in the feed and product respectively, will pass.
• Bond’s formula gives a fairly reliable first approximation to the energy requirement provided the product top size is not less than 100 μm

• In differential form Bond’s formula becomes:

\[
\frac{dE}{dx} = C_B \frac{1}{x^{3/2}}
\]

• It can be seen from the above analysis that the three proposals can be considered as being the integrals of the same differential equation:

\[
\frac{dE}{dx} = -C \frac{1}{x^N}
\]

- \(N = 2\) \(C = C_R\) for Rittinger
- \(N = 1\) \(C = C_K\) for Kick
- \(N = 1.5\) \(C = C_B\) for Bond
• It has been suggested that the three approaches to prediction of energy requirements are each more applicable in certain areas of product size
• It is common practice to assume that Kick’s proposal is applicable for large particle size (coarse crushing and crushing),
• Rittinger’s for very small particle size (ultra-fine grinding)
• Bond’s formula being suitable for intermediate particle size, the most common range for many industrial grinding processes
• It is common to model the breakage process in comminution equipment on the basis of two functions, the specific rate of breakage and the breakage distribution function

• The specific rate of breakage $S_j$ is the probability of a particle of size $j$ being broken in unit time

• The breakage distribution function $b(i,j)$ describes the size distribution of the product from the breakage of a given size of particle

• For example, $b(i,j)$ is the fraction of breakage product from size interval $j$ which falls into size interval $i$

• The breakage distribution function may also be expressed in cumulative form as $B(i,j)$, the fraction of the breakage product from size interval $j$ which falls into size intervals $j$ to $n$, where $n$ is the total number of size intervals

• $B(i,j)$ is thus a cumulative undersize distribution
• S is a rate of breakage
• The following equation expresses the rate of change of the mass of particles in size interval i with time:

\[
\frac{dm_i}{dt} = \sum_{j=1}^{j=i-1} [b(i,j)S_jm_j] - S_im_i
\]

• Where

\[
\sum_{j=1}^{j=i-1} [b(i,j)S_jm_j] = \text{mass broken into interval } i \text{ from all intervals of } j > i
\]

\[
S_im_i = \text{mass broken out of interval } i
\]

• Since \( m_i = y_iM \) and \( m_j = y_jM \), where M is the total mass of feed material and \( y_i \) is the mass fraction in size interval i
• We can write a similar expression for the rate of change of mass fraction of material in size interval i with time:

\[
\frac{dy_i}{dt} = \sum_{j=1}^{j=i-1} [b(i,j)S_jy_j] - S_iy_i
\]
• Thus, with a set of S and b values for a given feed material, the product size distribution after a given time in a mill may be determined

• In practice, both S and b are dependent on particle size, material and machine

• The specific rate of breakage should decrease with decreasing particle size

• Aim of this approach is to be able to use values of S and b determined from small-scale tests to predict product size distributions on a large scale

• This method is found to give fairly reliable predictions
WORKED EXAMPLE 12.1

A material consisting originally of 25 mm particles is crushed to an average size of 7 mm and requires 20 kJ/kg for this size reduction. Determine the energy required to crush the material from 25 mm to 3.5 mm assuming (a) Rittinger’s law, (b) Kick’s law and (c) Bond’s law.
Solution

(a) Applying Rittinger's law as expressed by Equation (12.4):

\[ 20 = C_R \left( \frac{1}{7} - \frac{1}{25} \right) \]

hence \( C_R = 194.4 \) and so with \( x_2 = 3.5 \text{ mm} \),

\[ E = 194.4 \left( \frac{1}{3.5} - \frac{1}{25} \right) \]

hence \( E = 47.8 \text{ kJ/kg} \)

(b) Applying Kick's law as expressed by Equation (12.7):

\[ 20 = -C_K \ln \left( \frac{7}{25} \right) \]

hence \( C_K = 15.7 \) and so with \( x_2 = 3.5 \text{ mm} \),

\[ E = -15.7 \ln \left( \frac{3.5}{25} \right) \]

hence \( E = 30.9 \text{ kJ/kg} \)

(c) Applying Bond's law as expressed by Equation (12.8a):

\[ 20 = C_B \left( \frac{1}{\sqrt{7}} - \frac{1}{\sqrt{25}} \right) \]
Table 12W2.1  Specific rates of breakage and breakage distribution function for the ball mill

<table>
<thead>
<tr>
<th>Size interval (μm)</th>
<th>212–150</th>
<th>150–106</th>
<th>106–75</th>
<th>75–53</th>
<th>53–37</th>
<th>37–0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interval no.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>$s_j$</td>
<td>0.7</td>
<td>0.6</td>
<td>0.5</td>
<td>0.35</td>
<td>0.3</td>
<td>0</td>
</tr>
<tr>
<td>$b(1, j)$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$b(2, j)$</td>
<td>0.32</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$b(3, j)$</td>
<td>0.3</td>
<td>0.4</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$b(4, j)$</td>
<td>0.14</td>
<td>0.2</td>
<td>0.5</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$b(5, j)$</td>
<td>0.12</td>
<td>0.2</td>
<td>0.25</td>
<td>0.6</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$b(6, j)$</td>
<td>0.12</td>
<td>0.2</td>
<td>0.25</td>
<td>0.4</td>
<td>1.0</td>
<td>0</td>
</tr>
</tbody>
</table>

hence $C_B = 112.4$ and so with $x_2 = 3.5 \text{ mm}$,

$$E = 112.4 \left( \frac{1}{\sqrt{3.5}} - \frac{1}{25} \right)$$

hence $E = 37.6 \text{ kJ/kg}$
WORKED EXAMPLE 12.2

Values of breakage distribution function \( b(i, j) \) and specific rates of breakage \( S_j \) for a particular material in a ball mill are shown in Table 12W2.1. To test the validity of these values, a sample of the material with the size distribution indicated in Table 12W2.2 is to be ground in a ball mill. Use the information in these tables to predict the size distribution of the product after one minute in the mill. (Note: \( S_j \) values in Table 12W2.1 are based on 1 minute grinding time.)

Solution

Applying Equation (12.12):

Change of fraction in interval 1

\[
\frac{dy_1}{dt} = 0 - S_1 y_1 = 0 - 0.7 \times 0.2 = -0.14
\]

Hence, new \( y_1 = 0.2 - 0.14 = 0.06 \)

Table 12W2.2  Feed size distribution

<table>
<thead>
<tr>
<th>Interval no. ((j))</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fraction</td>
<td>0.2</td>
<td>0.4</td>
<td>0.3</td>
<td>0.06</td>
<td>0.04</td>
<td>0</td>
</tr>
</tbody>
</table>
Change of fraction in interval 2

\[
\frac{dy_2}{dt} = b(2, 1)S_1y_1 - S_2y_2 \\
= (0.32 \times 0.7 \times 0.2) - (0.6 \times 0.4) \\
= -0.1952
\]

Hence new \( y_2 = 0.4 - 0.1952 = 0.2048 \)

Change of fraction in interval 3

\[
\frac{dy_3}{dt} = [b(3, 1)S_1y_1 + b(3, 2)S_2y_2] - S_3y_3 \\
= [(0.3 \times 0.7 \times 0.2) + (0.4 \times 0.6 \times 0.4)] - (0.5 \times 0.3) \\
= -0.012
\]

Hence, new \( y_3 = 0.3 - 0.012 = 0.288 \)

Similarly for intervals 4, 5 and 6:

\[
\begin{align*}
\text{new } y_4 &= 0.1816 \\
\text{new } y_5 &= 0.1429 \\
\text{new } y_6 &= 0.1227
\end{align*}
\]

Checking:

Sum of predicted product interval mass fractions = \( y_1 + y_2 + y_3 + y_4 + y_5 + y_6 = 1.000 \)

Hence product size distribution:

<table>
<thead>
<tr>
<th>Interval no. (j)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fraction</td>
<td>0.06</td>
<td>0.2048</td>
<td>0.288</td>
<td>0.1816</td>
<td>0.1429</td>
<td>0.1227</td>
</tr>
</tbody>
</table>