Week # 9
MR Chapter 7

• Tutorial #9
• MR #7.1, 7.2, 7.3, 7.4, 7.5, 7.6.

• To be discussed on March 24, 2020.
• By either volunteer or class list.

MARTIN RHODES (2008)
Introduction to Particle Technology, 2nd Edition.
Publisher John Wiley & Son, Chichester, West Sussex, England.
# Fluidization

## Fundamentals

- When a fluid is passed upwards through a bed of particles the pressure loss in the fluid due to frictional resistance increases with increasing fluid flow.
- A point is reached when the upward drag force exerted by the fluid on the particles is equal to the apparent weight of particles in the bed.
- At this point the particles are lifted by the fluid, the separation of the particles increases and the bed becomes fluidized.
- The force balance across the fluidized bed dictates that the fluid pressure loss across the bed of particles is equal to the apparent weight of the particles per unit area of the bed.

\[
\text{pressure drop} = \frac{\text{weight of particles} - \text{upthrust on particles}}{\text{bed cross-sectional area}}
\]
• For a bed of particles of density $\rho_p$, fluidized by a fluid of density $\rho_f$ to form a bed of depth $H$ and voidage $\varepsilon$ in a vessel of cross-sectional area $A$

\[ \Delta p = \frac{HA(1-\varepsilon)(\rho_p - \rho_f)g}{A} \]

\[ \Delta p = H(1-\varepsilon)(\rho_p - \rho_f)g \]

• Plot of fluid pressure loss across the bed vs superficial fluid velocity through the bed:

![Plot of fluid pressure loss across the bed vs superficial fluid velocity through the bed.](image)
• OA is the packed bed region
• Solid particles do not move relative to one another and their separation is constant
• Pressure loss vs fluid velocity relationship is described by the Carman-Kozeny equation and the Ergun equation
• BC is the fluidized bed region
• At point A, pressure loss rises above the value predicted
• This rise is more marked in small vessels and in powders which have been compacted to some extent before the test
• Associated with the extra force required to overcome wall friction and adhesive forces between bed and distributor
• Superficial fluid velocity at which the packed bed becomes a fluidized bed is known as the minimum fluidization velocity, $U_{mf}$
• Sometimes referred to as the velocity at incipient fluidization
• $U_{mf}$ increases with particle size and particle density and is affected by fluid properties
• To derive expression for $U_{mf}$, equate expression for pressure loss in a fluidized bed with pressure loss across a packed bed
• Applying the Ergun equation,

\[
(1 - \varepsilon)(\rho_p - \rho_f)g = 150 \frac{(1 - \varepsilon)^2 \mu U_{mf}}{\varepsilon^3 x_{sv}^2} + 1.75 \frac{(1 - \varepsilon) \rho_f U_{mf}^2}{\varepsilon^3 x_{sv}}
\]

\[
\frac{-\Delta p}{H} = 150 \frac{(1 - \varepsilon)^2 \mu U}{\varepsilon^3 x_{sv}^2} + 1.75 \frac{(1 - \varepsilon) \rho_f U^2}{\varepsilon^3 x_{sv}}
\]

\[
(1 - \varepsilon)(\rho_p - \rho_f)g = 150 \frac{(1 - \varepsilon)^2}{\varepsilon^3} \left( \frac{\mu^2}{\rho_f x_{sv}^3} \right) \left( \frac{U_{mf} x_{sv} \rho_f}{\mu} \right)
+ 1.75 \frac{(1 - \varepsilon)}{\varepsilon^3} \left( \frac{\mu^2}{\rho_f x_{sv}^3} \right) \left( \frac{U_{mf}^2 x_{sv}^2 \rho_f^2}{\mu^2} \right)
\]
• Where $Ar$ is the dimensionless number known as the Archimedes number

$$
(1 - \varepsilon)(\rho_p - \rho_f)g\left(\frac{G \varphi^3}{\mu^2}\right) = 150 \frac{(1 - \varepsilon)^2}{\varepsilon^3} Re_{mf} + 1.75 \frac{(1 - \varepsilon)}{\varepsilon^3} Re_{mf}^2
$$

\[
Ar = 150 \frac{(1 - \varepsilon)}{\varepsilon^3} Re_{mf} + 1.75 \frac{1}{\varepsilon^3} Re_{mf}^2
\]

• And $Re_{mf}$ is the Reynolds number at incipient fluidization

$$
Ar = \frac{\rho_f(\rho_p - \rho_f)g\varphi^3}{\mu^2}
$$

$$
Re_{mf} = \left(\frac{U_{mf} \varphi \rho_f}{\mu}\right)
$$

• In order to obtain a value of $U_{mf}$, we need to know the voidage of the bed at incipient fluidization, $\varepsilon = \varepsilon_{mf}$
• A typical value of $\varepsilon_{mf}$ is 0.4

$$Ar = 1406 \, Re_{mf} + 27.3 \, Re_{mf}^2$$

• Wen and Yu (1966) produced an empirical correlation for $U_{mf}$

$$Ar = 1652 \, Re_{mf} + 24.51 \, Re_{mf}^2$$

• This correlation is valid for spheres in the range $0.01 < Re_{mf} < 1000$ and is often expressed in the form:

$$Re_{mf} = 33.7 [(1 + 3.59 \times 10^{-5} \, Ar)^{0.5} - 1]$$

• For gas fluidization, the Wen and Yu correlation is often taken as most suitable for particles larger than 100 $\mu$m

• The correlation of Baeyens and Geldart (1974) is best for particles less than 100 $\mu$m

$$U_{mf} = \frac{(\rho_p - \rho_f)^{0.934} \cdot \sqrt{g \cdot 0.934 \cdot \nu_p^{1.8}}}{1110 \mu^{0.87} \rho_f^{0.056}}$$
Relevant Powder and Particle Properties

• The correct density for use in fluidization equations is the particle density
• Defined as the mass of a particle divided by its hydrodynamic volume
• Volume ‘seen’ by the fluid in its fluid dynamic interaction with the particle
• Includes the volume of all open and closed pores

\[
\text{particle density} = \frac{\text{mass of particle}}{\text{hydrodynamic volume of particle}}
\]

\[
\text{absolute density} = \frac{\text{mass of particle}}{\text{volume of solids material making up the particle}}
\]

• Bed density is also used in connection with fluidized beds

\[
\text{bed density} = \frac{\text{mass of particles in a bed}}{\text{volume occupied by particles and voids between them}}
\]
For example, 600 kg of powder is fluidized in a vessel of cross-sectional area 1 m$^2$ and achieves a bed height of 0.5 m. What is the bed density?

Mass of particles in the bed = 600 kg

Volume occupied by particles and voids = $1 \times 0.5 = 0.5$ m$^3$

Hence, bed density = $600/0.5 = 1200$ kg/m$^3$.

If the particle density of these solids is 2700 kg/m$^3$, what is the bed voidage?

Bed density $\rho_B$ is related to particle density $\rho_p$ and bed voidage $\varepsilon$ by Equation (7.12):

$$\rho_B = (1 - \varepsilon)\rho_p$$

Hence, voidage = $1 - \frac{1200}{2700} = 0.555$.

Figure 7.2 Hydrodynamic volume of a particle
• Another density often used is the bulk density, defined in a similar way to fluid bed density

\[
\text{bulk density} = \frac{\text{mass of particles}}{\text{volume occupied by particles and voids between them}}
\]

• The most appropriate particle size to use in equations relating to fluid-particle interactions is a hydrodynamic diameter

• For use in fluidization applications, starting from a sieve analysis the mean size of the powder is often calculated from

\[
\text{mean } x_p = \frac{1}{\sum m_i/x_i}
\]

• Where \(x_i\) is the arithmetic mean of adjacent sieves between which a mass fraction \(m_i\) is collected
Bubbling and Non-bubbling Fluidization

- Beyond the minimum fluidization velocity bubbles or particle-free voids may appear in the fluidized bed.

- At superficial velocities above the minimum fluidization velocity, fluidization may in general be either bubbling or non-bubbling.

- Some combinations of fluid and particles give rise to only bubbling fluidization and some combinations give only non-bubbling fluidization.
• Most liquid fluidized systems do not give rise to bubbling

• Gas fluidized systems give either only bubbling fluidization or non-bubbling fluidization beginning at $U_{mf}$, followed by bubbling fluidization as fluidizing velocity increases

• Non-bubbling fluidization is also known as particulate or homogeneous fluidization

• Bubbling fluidization is often referred to as aggregative or heterogeneous fluidization
Classification of Powders

- Geldart (1973) classified powders into four groups according to their fluidization properties at ambient conditions.
- The Geldart classification of powders is now used widely in all fields of powder technology.
- Group A – powders which when fluidized by air at ambient conditions give a region of non-bubbling fluidization beginning at $U_{mf}$, followed by bubbling fluidization as fluidizing velocity increases.
- Group B – powders which under these conditions give only bubbling fluidization.
- Group C – powders which are very fine, cohesive and are incapable of fluidization.
- Group D – large particles distinguished by their ability to produce deep spouting beds.
<table>
<thead>
<tr>
<th>Property</th>
<th>Group C</th>
<th>Group A</th>
<th>Group B</th>
<th>Group D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Most obvious characteristic</td>
<td>Cohesive, difficult to fluidize</td>
<td>Ideal for fluidization. Exhibits range of non-bubbling fluidization</td>
<td>Starts bubbling at $U_{mf}$</td>
<td>Coarse solids</td>
</tr>
<tr>
<td>Typical solids</td>
<td>Flour, cement</td>
<td>Cracking catalyst</td>
<td>Building sand</td>
<td>Gravel, coffee beans</td>
</tr>
<tr>
<td>Bed expansion</td>
<td>Low because of chanelling</td>
<td>High</td>
<td>Moderate</td>
<td>Low</td>
</tr>
<tr>
<td>De-aeration rate</td>
<td>Initially fast, then exponential</td>
<td>Slow, linear</td>
<td>Fast</td>
<td>Fast</td>
</tr>
<tr>
<td>Bubble properties</td>
<td>No bubbles—only channels</td>
<td>Bubbles split and coalesce. Maximum bubble size</td>
<td>No limit to size</td>
<td>No limit to size</td>
</tr>
<tr>
<td>Solids mixing</td>
<td>Very low</td>
<td>High</td>
<td>Moderate</td>
<td>Low</td>
</tr>
<tr>
<td>Gas backmixing</td>
<td>Very low</td>
<td>High</td>
<td>Moderate</td>
<td>Low</td>
</tr>
<tr>
<td>Spouting</td>
<td>No</td>
<td>No</td>
<td>Only in shallow beds</td>
<td>Yes, even in deep beds</td>
</tr>
</tbody>
</table>
• Since the range of gas velocities over which non-bubbling fluidization occurs in Group A powders is small, bubbling fluidization is the type most commonly encountered in gas fluidized systems in commercial use.

• The superficial gas velocity at which bubbles first appear is known as the minimum bubbling velocity $U_{mb}$.

• Premature bubbling can be caused by poor distributor design or protuberances inside the bed.

• Abrahamsen and Geldart (1980) correlated the maximum values of $U_{mb}$ with gas and particle properties using the following correlation:

$$U_{mb} = 2.07 \exp(0.716F) \left(\frac{x_p \rho_g^{0.06}}{\mu^{0.347}}\right)$$

• Where $F$ is the fraction of powder less than 45 μm.
• In Group A powders, $U_{mb} > U_{mf}$
• Bubbles are constantly splitting and coalescing and a maximum stable bubble size is achieved
• This makes for good quality, smooth fluidization

![Diagram of fluidization](image)

• In Groups B and D powders, $U_{mb} = U_{mf}$
• Bubbles continue to grow, never achieving a maximum size
• This makes for rather poor quality fluidization associated with large pressure fluctuations
• In Group C powders, the interparticle forces are large compared with the inertial forces on the particles
• Particles are unable to achieve the separation required to be totally supported by drag and buoyancy forces
• True fluidization does not occur
• Bubbles do not appear, instead the gas flow forms channels through the powder
• Since the particles are not fully supported by the gas, pressure loss across the bed is always less than apparent weight of the bed per unit cross-sectional area
• When the size of the bubbles is greater than about one-third of the diameter of the equipment their rise velocity is controlled by the equipment and they become slugs of gas.
• Slugging is attended by large pressure fluctuations and so it is generally avoided in large units since it can cause vibration to the plant.
• Slugging is unlikely to occur at any velocity if the bed is sufficiently shallow.
• Slugging will not occur provided the following criterion is satisfied:

\[
\left( \frac{H_{mf}}{D} \right) \leq \frac{1.9}{(\rho_p x_p)^{0.3}}
\]

• If the bed is deeper than this critical height, then slugging will occur when the gas velocity exceeds $U_{ms}$:

\[
U_{ms} = U_{mf} + 0.16(1.34D^{0.175} - H_{mf})^2 + 0.07(gD)^{0.5}
\]
Entrainment

• The term entrainment describes the ejection of particles from the surface of a bubbling bed and their removal from the vessel in the fluidizing gas
• Other terms such as ‘carryover’ and ‘elutriation’ are often used
• Entrainment of particles in an upward-flowing gas stream is a complex process
• Rate of entrainment and size distribution of entrained particles will in general depend on particle size and density, gas properties, gas velocity, gas flow regime and vessel diameter
• Prediction of entrainment from first principles is not possible and in practice an empirical approach must be adopted
• Define coarse particles as particles whose terminal velocities are greater than the superficial gas velocity \( U_T > U \)
• Fine particles are particles which \( U_T < U \)
• Region above fluidized bed surface is considered to be composed of several zones
• Freeboard – region between the bed surface and gas outlet
• Splash zone – region just above the bed surface in which coarse particles fall back down
• Disengagement zone – region above the splash zone in which the upward flux and suspension concentration of fine particles decreases with increasing height
• Dilute-phase transport zone – region above the disengagement zone in which all particles are carried upwards, particle flux and suspension concentration are constant with height
• Transport disengagement height (TDH) – height from the bed surface to the top of the disengagement zone
• Above TDH the entrainment flux and concentration of particles is constant
• From design point of view, in order to gain maximum benefit from the effect of gravity in the freeboard, gas exit should be placed above TDH
• Many empirical correlations for TDH are available in the literature

\[ \text{TDH} = 4.47d_{bvs}^{0.5} \]

• Where \( d_{bvs} \) is the equivalent volume diameter of a bubble at the surface
Empirical estimation of entrainment rates from fluidized beds

$\left(\frac{\text{instantaneous rate of loss}}{\text{of solids of size } x_i}\right) \propto \text{bed area} \times \left(\frac{\text{fraction of bed with}}{\text{size } x_i \text{ at time } t}\right)

\text{i.e. } R_i = -\frac{dt}{d} (M_B m_{Bi}) = K_{ih}^* A m_{Bi}$

Where $K_{ih}^*$ is the elutriation rate constant (the entrainment flux at height $h$ above the bed surface for the solids of size $x_i$, when $m_{Bi} = 1.0$), $M_B$ is the total mass of solids in the bed, $A$ is the area of bed surface and $m_{Bi}$ is the fraction of the bed mass with size $x_i$ at time $t$

For continuous operation, $m_{Bi}$ and $M_B$ are constant

$R_i = K_{ih}^* A m_{Bi}$

The total rate of entrainment, $R_T = \sum R_i = \sum K_{ih}^* A m_{Bi}$

The solids loading of size $x_i$ in the off-gases is $\rho_i = R_i / UA$

The total solids loading of the gas leaving the freeboard is $\rho_T = \sum \rho_i$
• For batch operation, the rates of entrainment of each size range, the total entrainment rate and the particle size distribution of the bed change with time

\[ -\Delta(m_{Bi}; M_B) = K_{ih} A m_{Bi} \Delta t \]

• Where \( \Delta(m_{Bi}; M_B) \) is the mass of solids in size range \( i \) entrained in time increment \( \Delta t \)

\[ \text{Total mass entrained in time } \Delta t = \sum_{i=1}^{k} [\Delta(m_{Bi}; M_B)] \]

• Mass of solids remaining in the bed at time

\[ t + \Delta t = (M_B)_t - \sum_{i=1}^{k} [\Delta(m_{Bi}; M_B)] \]

• Where subscript \( t \) refers to the value at time \( t \)

\[ \text{Bed composition at time } t + \Delta t = \frac{(m_{Bi}, M_B)_t}{(M_B)_t - \sum_{i=1}^{k} [\Delta(m_{Bi}; M_B)]} \]

• Solution of a batch entrainment problem proceeds by sequential application of above equations for required time period
• The elutriation rate constant $K_{ih}^*$ cannot be predicted from first principles
• Correlations are usually in terms of the carryover rate above TDH, $K_{i∞}^*$
• For particles $> 100 \, \mu m$ and $U > 1.2 \, m/s$

\[
\frac{K_{i∞}^*}{ρ_g U} = 23.7 \exp \left( -5.4 \frac{U_{Ti}}{U} \right)
\]

• For particles $< 100 \, \mu m$ and $U < 1.2 \, m/s$

\[
\frac{K_{i∞}^*}{ρ_g U} = \begin{cases} 
1.26 \times 10^7 \left( \frac{U^2}{g x_i ρ_p^2} \right)^{1.88} & \text{when } \left( \frac{U^2}{g x_i ρ_p^2} \right) < 3 \times 10^{-4} \\
4.31 \times 10^4 \left( \frac{U^2}{g x_i ρ_p^2} \right)^{1.18} & \text{when } \left( \frac{U^2}{g x_i ρ_p^2} \right) > 3 \times 10^{-4}
\end{cases}
\]
Applications of Fluidized Beds

- Physical processes include drying, mixing, granulation, coating, heating and cooling
- These processes take advantage of the excellent mixing capabilities of the fluidized bed
- One of the most important applications is to the drying of solids
- Fluidized beds are often used to cool particulate solids following a reaction
- Cooling may be by fluidizing air alone or by use of cooling water passing through tubes immersed in the bed
• Gas fluidized bed is also a good medium to carry out a chemical reaction involving a gas and a solid
• Gas-solid contacting is generally good
• Excellent solids circulation within the bed promotes good heat transfer between bed particles and fluidizing gas and between the bed and heat transfer surfaces immersed in the bed
• Gives rise to near isothermal conditions even when reactions are strongly exothermic or endothermic
• Good heat transfer gives rise to ease of control of the reaction
• Fluidity of the bed makes for ease of removal of solids from reactor
WORKED EXAMPLE 7.2

A powder having the size distribution given below and a particle density of 2500 kg/m³ is fed into a fluidized bed of cross-sectional area 4 m² at a rate of 1.0 kg/s.

<table>
<thead>
<tr>
<th>Size range number (i)</th>
<th>Size range (μm)</th>
<th>Mass fraction in feed</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10–30</td>
<td>0.20</td>
</tr>
<tr>
<td>2</td>
<td>30–50</td>
<td>0.65</td>
</tr>
<tr>
<td>3</td>
<td>50–70</td>
<td>0.15</td>
</tr>
</tbody>
</table>

The bed is fluidized using air of density 1.2 kg/m³ at a superficial velocity of 0.25 m/s. Processed solids are continuously withdrawn from the base of the fluidized bed in order to maintain a constant bed mass. Solids carried over with the gas leaving the vessel are collected by a bag filter operating at 100% total efficiency. None of the solids caught by the filter are returned to the bed. Assuming that the fluidized bed is well-mixed and that the freeboard height is greater than the transport disengagement height under these conditions, calculate at equilibrium:

(a) the flow rate of solids entering the filter bag;

(b) the size distribution of the solids in the bed;

(c) the size distribution of the solids entering the filter bag;

(d) the rate of withdrawal of processed solids from the base of the bed;

(e) the solids loading in the gas entering the filter.

Solution

First calculate the elutriation rate constants for the three size ranges under these conditions from the Zenz and Weil correlation [Equation (7.46)]. The value of particle size $x$ used in the correlation is the arithmetic mean of each size range:

$x_1 = 20 \times 10^{-6} \text{ m}; \quad x_2 = 40 \times 10^{-6} \text{ m}; \quad x_3 = 60 \times 10^{-6} \text{ m}$
With \( U = 0.25 \text{ m/s}, \rho_p = 2500 \text{ kg/m}^3 \) and \( \rho_f = 1.2 \text{ kg/m}^3 \)

\[
\begin{align*}
K_{1\infty}^* &= 3.21 \times 10^{-2}\text{kg/m}^2\text{s} \\
K_{2\infty}^* &= 8.74 \times 10^{-3}\text{kg/m}^2\text{s} \\
K_{3\infty}^* &= 4.08 \times 10^{-3}\text{kg/m}^2\text{s}
\end{align*}
\]

Referring to Figure 7.W2.1 the overall and component material balances over the fluidized bed system are:

Overall balance: \( F = Q + R \)  
(7W2.1)

Component balance: \( Fm_{Fi} = Qm_{Qi} + Rm_{Ri} \)  
(7W2.2)

where \( F, Q \) and \( R \) are the mass flow rates of solids in the feed, withdrawal and filter discharge, respectively, and \( m_{Fi}, m_{Qi} \) and \( m_{Ri} \) are the mass fractions of solids in size range \( i \) in the feed, withdrawal and filter discharge, respectively.

From Equation (7.39) the entrainment rate of size range \( i \) at the gas exit from the freeboard is given by:

\[
R_i = Rm_{Ri} = K_{i\infty}^* A m_{Bi}
\]  
(7W2.3)

and

\[
R = \sum R_i = \sum Rm_{Ri}
\]  
(7W2.4)
Combining these equations with the assumption that the bed is well mixed \( (m_Q = m_B) \),

\[
m_B = \frac{F m_{F_i}}{F - R + K_{dc} A}
\]

(7W2.5)
Now both $m_{B_i}$ and $R$ are unknown. However, noting that $\sum m_{B_i} = 1$, we have

\[
\frac{1.0 \times 0.2}{1.0 - R + (3.21 \times 10^{-2} \times 4)} + \frac{1.0 \times 0.65}{1.0 - R + (8.74 \times 10^{-3} \times 4)} + \frac{1.0 \times 0.15}{1.0 - R + (4.08 \times 10^{-3} \times 4)} = 1.0
\]

Solving for $R$ by trial and error, $R = 0.05$ kg/s

(b) Substituting $R = 0.05$ kg/s in Equation (7W2.5), $m_{B_1} = 0.1855$, $m_{B_2} = 0.6599$ and $m_{B_3} = 0.1552$

Therefore size distribution of bed:

<table>
<thead>
<tr>
<th>Size range number ($i$)</th>
<th>Size range ($\mu$m)</th>
<th>Mass fraction in bed</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10–30</td>
<td>0.1855</td>
</tr>
<tr>
<td>2</td>
<td>30–50</td>
<td>0.6599</td>
</tr>
<tr>
<td>3</td>
<td>50–70</td>
<td>0.1552</td>
</tr>
</tbody>
</table>

(c) From Equation (7W2.3), knowing $R$ and $m_{B_i}$, we can calculate $m_{R_i}$:

\[
m_{R_i} = \frac{K_{1x} A m_{B_i}}{R} = \frac{3.21 \times 10^{-2} \times 4 \times 0.1855}{0.05} = 0.476
\]

similarly, $m_{R_2} = 0.4614$, $m_{R_3} = 0.0506$

Therefore size distribution of solids entering filter:

<table>
<thead>
<tr>
<th>Size range number ($i$)</th>
<th>Size range ($\mu$m)</th>
<th>Mass fraction entering filter</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10–30</td>
<td>0.476</td>
</tr>
<tr>
<td>2</td>
<td>30–50</td>
<td>0.4614</td>
</tr>
<tr>
<td>3</td>
<td>50–70</td>
<td>0.0506</td>
</tr>
</tbody>
</table>

(d) From Equation (7W2.1), the rate of withdrawal of solids from the bed, $Q = 0.95$ kg/s

(e) Solids loading for gas entering the filter,

\[
\frac{\text{mass flow of solids}}{\text{volume flow of gas}} = \frac{R}{UA} = 0.05 \text{ kg/m}^3
\]