Forecasting Singapore’s quarterly GDP with monthly external trade

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Abstract

In this paper we suggest a methodology to formulate a dynamic regression with variables observed at different time intervals. This methodology is applicable if the explanatory variables are observed more frequently than the dependent variable. We demonstrate this procedure by developing a forecasting model for Singapore’s quarterly GDP based on monthly external trade. Apart from forecasts, the model provides a monthly distributed lag structure between GDP and external trade, which is not possible with quarterly data. © 1998 Elsevier Science B.V. All rights reserved.

Keywords: Variables observed at different time intervals; Non-linear dynamic regression

1. Introduction

This paper has a twofold objective. One is to provide a model which can relate variables observed at different time intervals such as quarterly and monthly. The standard approach when variables are observed at different time intervals is to change them to a common time interval through temporal aggregation or systematic sampling depending on whether the variables are flow variables or stock variables respectively. This approach, apart from losing information, may defeat the purpose of using a readily available variable to forecast a key variable of interest. Moreover, the undesirable effects of temporal aggregation and systematic sampling on the model structure, parameter estimates, forecasting and causal relationships are well documented (Wei, 1990).

The second objective is to use the methodology to construct a model to forecast Singapore’s quarterly GDP growth using monthly external trade figures. We will show how the highly volatile growth rates of external trade are effectively smoothened by the model to produce reasonably accurate forecasts of GDP growth rates.

Section 2 presents a transformation which connects two variables observed at different frequencies. Sections 3 and 4 deal with the application. We subjected every fitted model to a number of diagnostic tests and for brevity these results are summarized in Appendix (Table A1).

2. Methodology

The procedure to be discussed is applicable when the dependent variable is observed less frequently compared to independent variables. For example, the dependent variable may be observed quarterly and the independent variables monthly. The objective is to incorporate the monthly variables directly in the
model without aggregating or systematically sampling them to quarterly values to match the dependent variable. The basic methodology presented here evolved from the ideas contained in Dhuvarajan (1976); Weiss (1984); Wei (1990).

We present a simple case first which can easily be generalised. Consider the model

$$y_t = \beta_0 + \beta_1 y_{t-\tau} + \lambda y_{t-\tau} + u_t, \quad t = 1, 2, 3, \ldots, T$$

(1)

where $\tau$ is a fraction of the time interval at which $y$ is observed. If $y$ is observed annually and $x$ is observed quarterly then $\tau = 1/4$. For quarterly and monthly variables $\tau = 1/3$. Note that $y_t$ is observed only at integer values $t = 1, 2, \ldots, T$. $u_t$ is assumed to be a white noise process with zero mean and constant variance.

Model Eq. (1) cannot be estimated as it is because $y_{t-\tau}$ is not observed. To get rid of this fractional lag, lag Eq. (1) by $\tau t$ and multiply by $\lambda^r$ and sum over the range of $r$. This gives

$$\sum \lambda^r y_{t-r\tau} = \sum \lambda^r \beta_0 + \beta_1 \sum \lambda^r x_{t-r\tau}$$

$$+ \sum \lambda^{r+1} y_{t-(r+1)\tau} + \sum \lambda^r u_{t-r\tau},$$

$$r = 0, 1, \ldots, 1/\tau - 1.$$  

(2)

After cancellation of terms on the LHS and RHS of Eq. (2) we get

$$y_t = \alpha + \beta_1 z_t + \lambda^{1/\tau} y_{t-1} + v_t, \quad t = 1, 2, \ldots, T$$

(3)

where

$$z_t = \sum_{r=0}^{1/\tau-1} \lambda^r x_{t-r\tau}, \quad v_t = \sum_{r=0}^{1/\tau-1} \lambda^r u_{t-r\tau},$$

$$\alpha = \beta_0 \sum_{r=0}^{1/\tau-1} \lambda^r.$$ .

What model Eq. (3) does is to convert the fractional lag in Eq. (1) into an integer lag with $z_t$ made up of a weighted sum of $x$’s which are observed at fractional lags. For example, if $y$ is quarterly and $x$ is monthly then $z_t$ is the weighted sum of the three months of that quarter. Furthermore, $v_t$ satisfies the classical assumptions at integer lags.

Model Eq. (3) is more meaningful when the dependent variable is a stock variable. For a flow variable, however, $y$ may be viewed as the period average. If $y$ is quarterly and $x$ is monthly then $y$ may be viewed as the average value per month. Empirically, however, we can use the quarterly values of $y$ directly in the model without having to divide them by 3 to obtain the monthly average. It should be noted that model Eq. (3) is non-linear in parameters because of $\lambda$. If $\lambda$ is known, the model is linear. If not then a non-linear estimation method is needed to estimate the model.

Model Eq. (1) can be easily extended to a more general distributed lag model by introducing additional lag terms of $x$ into the model. It is not difficult to verify that adding additional lag terms of $x$ into Eq. (1) will amount to adding additional lag terms of $z$ into Eq. (3). Therefore, a more general model takes the form

$$y_t = \alpha + \beta_1 z_t + \beta_2 z_{t-2\tau} + \beta_3 z_{t-3\tau} + \cdots$$

$$+ \beta_m z_{t-m\tau} + \lambda^{1/\tau} y_{t-1} + v_t, \quad t = 1, 2, 3, \ldots, T$$

(4)

where $\alpha = \alpha_0 (1 + \lambda + \lambda^2 + \cdots + \lambda^{1/\tau-1})$. $\alpha_0$ is the intercept term of the original model and $m$ is an integer representing the lag length. The distributed lag structure at the time intervals of $\tau$ can be worked out by substituting $x$ into $z$ in Eq. (4) and by removing the autoregressive (AR) lag through expansion.

Any number of explanatory variables can also be used in model Eq. (1) but they will all take the form of $z$. One AR lag is the easiest to handle within the above modelling framework. However, the model can entertain more than one AR lag with some extra modelling effort. This will increase the non-linearities of the model.

3. A forecasting model for Singapore’s GDP growth

External demand has been the driving force of Singapore’s economic growth. However, data needed to construct an index of external demand become available only with a substantial time lag. An intermediate variable which reflects external demand is Singapore’s total external trade (merchandise exports plus merchandise imports). There is a growing literature which highlights a direct relationship
between external trade and economic growth. Levine and Renelt (1992) have found that what matters is total trade not just exports alone. Furthermore, the trade figures are promptly available on a monthly basis whereas GDP figures are available on a quarterly basis but with a lag of almost one quarter. In this section we will apply the methodology of the previous section to construct a model by relating quarterly GDP to monthly external trade (TRADE).

Although the movement of external trade in Singapore is taken as a very important barometer for subjective forecasting of GDP growth, none has attempted to model TRADE and GDP directly primarily because the TRADE growth rates tend to be far more volatile than the GDP growth rates. For example, in 1991Q4 trade growth fell to \(-8.3\%\) whereas the GDP recorded a 6.1\% growth. We will see that the use of the monthly trade figures in our model provides an effective smoothing filter for forecasting.

Some preliminaries pertaining to the modelling GDP and TRADE are given below. Both series contain a seasonal component. Although GDP may be characterised by a seasonal unit root process (Abeysinghe, 1994a) TRADE contains only a stationary seasonal component. The use of seasonal dummies in general tend to produce poor forecasts (Abeysinghe, 1994b). Therefore, it is difficult to model seasonality directly within the present framework. The approach we adopt is to use seasonally adjusted series (see Appendix A). As pointed out by Sims (1993); Hansen and Sargent (1993) it is advisable to use seasonally adjusted data instead of mis-specifying the seasonal.

Fig. 1 shows the seasonally adjusted quarterly series converted to a common base of 100 in 1975Q1 for easy comparison of trends. The two series have different trends and they cannot be cointegrated. This is also confirmed by a residual-based Augmented Dickey-Fuller (ADF) test. Although trade is the driving force of Singapore's economic growth the total trade figures constitute gross values whereas GDP is a value-added concept, therefore the lack of cointegration is not surprising. To remove the trend we use the first difference of the logarithm of the variables. Note that for GDP the differencing is on a quarter-on-quarter basis and for trade it is on a month-on-month basis.

In this exercise we use the data over 1975 to 1992 for model estimation and 1993–1996 for model evaluation. In 1993 the Singapore economy took a sharp upturn surpassing all expectations of a steady slowing down of GDP growth. This is the basis for the break-up of the sample period into the estimation and validation periods.

Let \(y_t\) denote the first difference of log of quarterly GDP and \(x_t\) denote the first difference of log of monthly TRADE. After a search we arrived at the following model and estimates using a non-linear LS method:

![Fig. 1. GDP and TRADE (Index No. 1975Q1 = 100).](image-url)
\[ y_t = 0.008(1 + \lambda + \lambda^2) + 0.069z_t + 0.049z_{t-1/3}^{1/3} + 0.077z_{t-2/3}^{1/3} + 0.091z_{t-1} + \lambda^3y_{t-1} + \nu_t, \]

where

\[ z_t = x_t + \lambda x_{t-1/3} + \lambda^2 x_{t-2/3}, \]

\[ t = \frac{1}{3}, \frac{2}{3}, 1, \frac{2}{3}, 2, \ldots, T \]

\[ \lambda = 0.388. \]  

The numbers in parentheses below Eq. (5) are t statistics. The fitted model passes through a number of commonly used diagnostic tests at the 5% level of significance (see Table A1).

3.1. Lag effects of external trade on GDP

If \( z_t \) is substituted with \( x_t \), we can see that model Eq. (5) uses trade data over six months. The right hand side of Eq. (5) can be expressed entirely in terms of the \( x_t \)'s which will show the complete distributed lag relationship between the growth rates of GDP and external trade. Table 1 shows the lag effects over a 12 month period and Fig. 2 plots the monthly lag effect of a 1% growth of TRADE on GDP growth.

Two important features that emerge from Table 1 and Fig. 2 are: (a) the inverted U shape of the lag distribution and (b) about 96% of the lag effect is complete within 12 months. The highest lag effect occurs on the fourth month. Such a lag pattern will not be revealed by aggregated quarterly data.

Based on these calculations we can conclude that a 1% continued growth in TRADE will lead to 0.69% growth in GDP within 12 months. However, the past growth rates of GDP and TRADE show that a substantial drop in TRADE growth does not necessarily produce a similar drop in GDP growth. This is because of the interference of the other factors, mainly domestic, which could offset the negative impact of trade on growth. The construction sector in Singapore in particular shows a counter-cyclical growth pattern. Therefore, as a rule of thumb, we can say that a 10% growth in external trade will produce 6–7% growth in GDP, given other things remain the same.

3.2. Post-sample forecast evaluation

In this section we compare the forecasting performance of model Eq. (5) against two alternatives, an ARIMA model and a regression model based on (aggregated) quarterly data. Again using data over the period 1975Q1–1992Q4 the univariate model we could select for \( \Delta \ln(GDP) \) on the basis of Akaike Information Criterion and residual autocorrelation checks was an AR(1). The best model we could fit the quarterly GDP and TRADE data is:

\[ \Delta \ln GDP_t = 0.0143 + 0.1416 \Delta \ln TRADE_t, \]

\[ R^2 = 0.26 \]  

### Table 1

<table>
<thead>
<tr>
<th>Month</th>
<th>Lag effect</th>
<th>Cumulative effect</th>
<th>Cumulative effect %</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.069</td>
<td>0.069</td>
<td>9.6</td>
</tr>
<tr>
<td>1</td>
<td>0.076</td>
<td>0.145</td>
<td>20.1</td>
</tr>
<tr>
<td>2</td>
<td>0.106</td>
<td>0.257</td>
<td>34.9</td>
</tr>
<tr>
<td>3</td>
<td>0.155</td>
<td>0.406</td>
<td>56.5</td>
</tr>
<tr>
<td>4</td>
<td>0.076</td>
<td>0.483</td>
<td>67.1</td>
</tr>
<tr>
<td>5</td>
<td>0.055</td>
<td>0.538</td>
<td>74.7</td>
</tr>
<tr>
<td>6</td>
<td>0.060</td>
<td>0.598</td>
<td>83.1</td>
</tr>
<tr>
<td>7</td>
<td>0.030</td>
<td>0.628</td>
<td>87.2</td>
</tr>
<tr>
<td>8</td>
<td>0.021</td>
<td>0.649</td>
<td>90.2</td>
</tr>
<tr>
<td>9</td>
<td>0.023</td>
<td>0.672</td>
<td>93.4</td>
</tr>
<tr>
<td>10</td>
<td>0.012</td>
<td>0.684</td>
<td>95.0</td>
</tr>
<tr>
<td>11</td>
<td>0.008</td>
<td>0.692</td>
<td>96.2</td>
</tr>
<tr>
<td>Total long run effect</td>
<td>–</td>
<td>0.720</td>
<td>100</td>
</tr>
</tbody>
</table>
Unlike model Eq. (5), model Eq. (6) fails both ARCH and heteroscedasticity tests at the 5% level (see Table A1). Curiously the lagged dependent variables in model Eq. (6) turn out to be statistically insignificant.

Table 2 shows the root mean square errors (RMSE) and mean absolute percentage errors (MAPE) of GDP forecasts by the three models. What we observe in Table 2 is that although the quarterly-monthly model Eq. (5) provides improved forecasts, the gains are marginal compared to the alternatives. It may be the case that the particular application we have chosen does not show up the full potential of the methodology. It is worth noting, however, that model Eq. (5) provides better longer term forecasts compared to the other two. If we take the one-quarter ahead forecast RMSE of model Eq. (5) as the base (349.4) then the 8-quarter ahead RMSE of model Eq. (5) is 64% larger than the base, whereas that for model Eq. (6) is 92% larger than the base. For the AR(1) model this number is 125%. As we have seen in the previous section, an additional advantage of model Eq. (5) is that it enables us to work out the monthly distributed lag structure which is not possible with aggregated quarterly data.

4. Supplementary models

The major advantage of model Eq. (5) presented above is that as monthly trade data become available the one-quarter-ahead GDP forecast can be updated...
quickly. However, for longer term forecasting of GDP growth we have to forecast TRADE first. We have developed the following model for this purpose:

\[ \text{TRADE} = X + M, \quad M = f(X), \]

\[ X = f(Y^f, \text{USORDERS}) \]

where \( X = \text{exports}, \ M = \text{imports}, \ Y^f = \text{a measure of the income of Singapore’s major trading partners}, \) and \( \text{USORDERS} = \text{new orders of electronics in the USA}. \) As our focus is on the ready availability of monthly data we use industrial production indexes to measure \( Y^f. \) Given the heavy dependence of Singapore’s manufacturing sector on electronics exports to the US, USORDERS stands primarily as a leading indicator. Singapore’s imports are primarily driven by exports. We find that the two series are strongly correlated and cointegrated. Therefore, instead of modelling exports and imports separately we relate total trade to external demand \( (Y^f) \) and USORDERS. This also reduces the compounding of estimation errors.

To measure \( Y^f \) we use the industrial production indexes (IPI) of the US, Japan and Malaysia. Singapore’s total trade with these three countries has amounted to more than 60% during the 1984–1994 period. Although Hong Kong is the other important trading partner an IPI on monthly basis is not available for Hong Kong. Using average trade weights over 1984–1994, \( Y^f \) is calculated as

\[ \ln Y^f = 0.38 \ln \text{IPI}_{\text{US}} + 0.31 \ln \text{IPI}_{\text{Japan}} + 0.31 \ln \text{IPI}_{\text{Malaysia}}. \]

A plot of the growth rates of TRADE and USORDERS shows that a clear leading relationship of USORDERS with TRADE emerges since about 1987. For this reason we restrict the analysis to the period 1987M1–1996M12. Although the three series (TRADE, \( Y^f, \) and USORDERS) show a close co-movement, tests do not provide conclusive evidence in support of cointegration. For brevity we do not report all the results here. In short we find that the individual series are I(1) and both the residual based ADF test and Johansen test give contradictory results on cointegration depending on the lag length chosen. For this reason we fitted two models, one assuming cointegration (an error correction model) and the other assuming no-cointegration, and selected the one which provided the best post-sample forecasts. The final versions of the models were decided after a considerable search. The results are given below.

**ECM assuming cointegration:**

\[
\Delta \ln \text{TRADE}_t = 1.37 - 0.37 \Delta \ln \text{TRADE}_{t-1} + 1.64 \Delta \ln Y^f_{t-3} + 0.42 \Delta \ln \text{USORDERS}_{t-5} - 0.43 \ln \text{TRADE}_{t-1} - 1.41 \ln Y^f_{t-4} - 0.73 \ln \text{USORDERS}_{t-6},
\]

\[ R^2 = 0.45 \]  

(7)

**Model assuming no-cointegration:**

\[
\Delta \ln \text{TRADE}_t = 0.007 - 0.55 \Delta \ln \text{TRADE}_{t-1} + 0.89 \Delta \ln Y^f_{t-3} + 1.35 \Delta \ln Y^f_{t-3} + 0.28 \Delta \ln \text{USORDERS}_{t-5},
\]

\[ R^2 = 0.38 \]  

(8)

Model Eq. (7) was estimated as in Bärßon (1989). All diagnostic statistics for both models, except for the normality test, are satisfactory (see Table A1). Table 3 gives a post-sample forecast comparison of the two models over 1993M1–1996M12. The table provides mean absolute percentage errors (MAPE) and \( R^2 \) between the actual and predicted values. Although one may be tempted to select the ECM (Model Eq. (7)) because of its better in-sample fit and because of its appeal as an ECM, the post-sample statistics are invariably in favour of the non-ECM model. We, therefore, use model Eq. (8) in our forecasting exercise. The four exogenous variables were fitted with AR models: \( \Delta \ln \text{IPI}_{\text{US}} \sim \text{AR}(2), \Delta \ln \text{IPI}_{\text{Japan}} \sim \text{AR}(3), \Delta \ln \text{IPI}_{\text{Malaysia}} \sim \text{AR}(2), \) and \( \Delta \ln \text{USORDERS} \sim \text{AR}(2). \)

The above set of equations (models Eq. (5), Eq. (8) and the AR models), which we refer to as the GDP-TRADE model, constitutes the complete spe-
Table 3
Post-sample forecasting performance of TRADE models

<table>
<thead>
<tr>
<th>Months</th>
<th>$n$</th>
<th>ECM (Model Eq. (7))</th>
<th>Non ECM (Model Eq. (8))</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>MAPE</td>
<td>$R^2$</td>
</tr>
<tr>
<td>1</td>
<td>48</td>
<td>4.48</td>
<td>0.84</td>
</tr>
<tr>
<td>3</td>
<td>46</td>
<td>5.69</td>
<td>0.69</td>
</tr>
<tr>
<td>6</td>
<td>43</td>
<td>6.61</td>
<td>0.54</td>
</tr>
<tr>
<td>9</td>
<td>40</td>
<td>7.00</td>
<td>0.37</td>
</tr>
<tr>
<td>12</td>
<td>37</td>
<td>7.03</td>
<td>0.18</td>
</tr>
</tbody>
</table>

cification of the model. Fig. 3 plots the actual and forecast quarterly GDP growth rates from 1993Q1 onwards derived from the full model estimated using data up to 1992M12. The forecast growth rates are based on four-quarter-ahead forecasts. Although the forecasts track the turning points reasonably well the model requires further improvements to obtain more accurate forecasts. As in many forecasting exercises, the forecasts of the exogenous variables may have to go through a subjective adjustment to reflect anticipated events.

In general, annual forecasts tend to be more accurate than the quarterly ones because the averaging cancels out the errors. For comparison Table 4 presents two alternative forecasts which are frequently quoted in Singapore. One is announced by the Econometric Studies Unit (ESU) of the National University of Singapore and the other by the Ministry of Trade and Industries (MTI). The ESU model consists of more than 80 equations including identities. The nature of MTI model is unknown. However, the MTI model has access to policy information (e.g. foreign work force in Singapore) which is not available to others. The RMSEs in Table 4 show that the pure mechanical forecasts generated by

<table>
<thead>
<tr>
<th>Year</th>
<th>Actual</th>
<th>GDP-TRADE model</th>
<th>ESU</th>
<th>MTI</th>
</tr>
</thead>
<tbody>
<tr>
<td>1993</td>
<td>10.4</td>
<td>9.4</td>
<td>6.4</td>
<td>6–7</td>
</tr>
<tr>
<td>1994</td>
<td>10.5</td>
<td>11.1</td>
<td>8.5</td>
<td>6–8</td>
</tr>
<tr>
<td>1995</td>
<td>8.8</td>
<td>7.0</td>
<td>8.1</td>
<td>7.5–8.5</td>
</tr>
<tr>
<td>1996</td>
<td>7.0</td>
<td>6.4</td>
<td>8.8</td>
<td>7.5–8.5</td>
</tr>
<tr>
<td>RMSE</td>
<td>–</td>
<td>1.1</td>
<td>2.4</td>
<td>3.2–2.2</td>
</tr>
</tbody>
</table>

Table 4
A comparison of GDP growth forecasts made at the beginning of the year

Fig. 3. Actual and forecast GDP growth rates.
our GDP-TRADE model has much to deserve considering the fact that the other two utilize more information and expert opinion.

5. Conclusion

We have outlined a procedure to construct a dynamic regression when the explanatory variables are observed more frequently than the dependent variable. This procedure preserves the information content of the explanatory variables, which would otherwise be lost in the standard approach of temporal aggregation and systematic sampling. Further effort is needed to generalise this procedure to more complicated model structures.

We demonstrated the procedure with a simple model constructed to forecast Singapore’s quarterly GDP with monthly external trade. Our quarterly-monthly data model provides reasonably good forecasts over a longer period compared to a number of alternative models considered in the paper. Moreover, our model can provide the monthly distributed lag relationship between GDP and TRADE, which is not possible with a model based on aggregated quarterly data.

This modelling strategy may also be very useful in forecasting the exogenous variables in large macroeconometric models. Such exogenous variables usually undergo a subjective forecasting. Suppose a key exogenous variable in the model is a quarterly GDP index of the major trading partners. Forecasts of this index can be made more objectively based on our procedure using monthly industrial production indexes which become available more quickly than the GDP figures.

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Appendix A

A.1. Data sources

GDP at 1990 prices and total external trade at current prices are from the data base TREND, Department of Statistics, Singapore. Industrial Production Indexes are from the IMF IFS data bank. These are in 1990 base year. US new orders of electronics (in value terms) are from Manufacturer’s Shipments, Inventories and Orders, US Department of Commerce. The US and Japan industrial production indexes were available in seasonally adjusted form. All other series were seasonally adjusted using X11-ARIMA procedure.

<table>
<thead>
<tr>
<th>Diagnostic test statistics for fitted models</th>
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<tbody>
<tr>
<td>Null hypothesis</td>
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<tr>
<td>----------------------------------</td>
</tr>
<tr>
<td>No residual autocorrelation</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>No autoregressive conditional</td>
</tr>
<tr>
<td>Heteroscedasticity (ARCH) in errors</td>
</tr>
<tr>
<td>No heteroscedasticity in errors</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Normally distributed errors</td>
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<td></td>
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<tr>
<td>Correct functional form (RESET)</td>
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<td></td>
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<tr>
<td>Stable parameters (Chow test)</td>
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<td></td>
</tr>
</tbody>
</table>

* Indicates the rejection of the null hypothesis at the 5% level of significance.

Notes: The test statistics were computed using PCGIVE 8.0. For more details, see Doornik and Hendry, 1995, Section 22.5). The normality test is a Chi-square test. All other test statistics have an F distribution. The degrees of freedom are omitted as they differ across tests and models. Instead, the P-values are given in parentheses. The estimation period is 1975–1992. The parameter stability test is over the forecast horizon 1993–1996.

References


**Biography:** Tilak ABEYSINGHE is a senior lecturer in econometrics in the Department of Economics and Statistics of the National University of Singapore (NUS). He has been an active member of the Econometric Studies Unit of NUS since 1992 and has developed a large-scale macroeconometric model for the Singapore economy. He is also a joint editor of Singapore Economic Review. His research publications have covered a wide range of topics such as experimental agriculture, population economics, international economics, econometrics and time series analysis.