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Structural Equation Modeling

- *Structural equation modeling* (SEM) has become an important statistical tool in behavioral sciences.

- SEM can be used to test multivariate variables with latent relationships.

- Confirmatory factor analysis, path analysis and regression are some special cases of it.

- When the number of studies increases, there is a need to synthesize these studies *meta-analytically.*
Meta-analytic Structural Equation Modeling

- Conventional two-stage approach for meta-analytic SEM:
  - **Stage 1**: *Are all the studies homogeneous?*
    - Pool correlation matrices with Hunter-Schmidt (H-S) approach (Hunter & Schmidt, 1990) or Hedges-Olkin (H-O) approach (Hedges & Olkin, 1985);
  - **Stage 2**: *Do the proposed models fit the data?*
    - Fit proposed models with the pooled correlation matrix with SEM software like LISREL and EQS.
Potential Problems of the Conventional Approaches

- **Stage 1**: pairwise aggregation of correlation coefficients with missing values across different studies

- **Stage 2**:
  - The resultant correlation matrix for input is not positive definite because of the pairwise aggregation;
  - Different researchers use different sample sizes, e.g., arithmetic and harmonic means, median or total (Viswesvaran & Ones, 1995). This makes the statistical inferences arbitrary;
  - Correlation matrix is used as covariance matrix. This makes the statistical inferences questionable.
Proposed Two-stage Structural Equation Modeling Approach

Based on multi-sample SEM and the analysis of correlation structure, a two-stage structural equation modeling (TSSEM) approach was proposed.

Stage 1: the homogeneity of correlation matrices is tested and the pooled correlation matrix is estimated:

$$
\Sigma(\theta)^g = \Lambda^g \Phi^g \Lambda^g + \Psi^g
$$

where $\Lambda^g$ is a diagonal matrix, $\text{Diag}[\Phi^g]$=Identity and $\Psi^g$ =0.

1) Suitable cross-group constraints can be applied on $\Phi^g$ ;
2) The estimated $\hat{\Phi}$ is the pooled correlation matrix;
**Stage 2**: the pooled correlation matrix can be used to test models with arbitrary distribution free (ADF) method:

\[ F(\theta) = (r - \rho(\theta))^T W^{-1} (r - \rho(\theta)) \]

- where \( W \) is the appropriate weight matrix, and \( r \) and \( \rho(\theta) \) are vec\(u\)(\(R\)) and vec\(u\)(\(P(\theta)\)), respectively.

- 1) The asymptotic covariance matrix of from Stage 1 is used as \( W \);
- 2) The total sample size from Stage 1 is used as the sample size for model fitting.
Some Simulation Results from Cheung (2002)

- **Stage 1**: the parameter estimates and rejection frequency of H-S, H-O and TSSEM are similar.

- **Stage 2**:
  - 1) Chi-square test statistics: H-S and H-O over-estimate a lot. Thus, they over-reject the true models seriously. TSSEM follows the expected chi-square distribution approximately; and
  - 2) The standard errors of TSSEM are accurate while H-S and H-O do not.
A Real Example

Data from International Social Survey Program (1989).

- 9 work-related variables
- 3 factor model
- 11 countries
- 7,155 participants
Stage 1 Results: Testing the homogeneity of correlation matrices

1) **H-S and H-O**: all smaller than .001

2) **TSSEM**: $\chi^2(360, N=7,155) = 941.42$, $p < .001$, CFI = 0.9428, NNFI = 0.9371, RMSEA = 0.0497 and SRMR = 0.0798.

**Interpretation**: the significant results of chi-square test may be just caused by the large sample size. The goodness-of-fit indices give us support that the correlation matrices are quite homogeneous.
Stage 2 Results: Fitting CFA models

1) **H-S**: $\chi^2(24, N=7,155) = 1,622$, $p < .001$, $CFI = 0.8340$, $NNFI = 0.7510$, $RMSEA = 0.1019$ and $SRMR = 0.0613$

2) **H-O**: $\chi^2(24, N=7,155) = 1,607$, $p < .001$, $CFI = 0.8321$, $NNFI = 0.7509$, $RMSEA = 0.1015$ and $SRMR = 0.0610$

3) **TSSEM**: $\chi^2(24, N=7,155) = 1,277$, $p < .001$, $CFI = 0.8671$, $NNFI = 0.8007$, $RMSEA = 0.0854$, $SRMR = 0.0683$.

The Chi-squares of H-S and H-O are 25% larger than that of TSSEM. TSSM fits better generally.
Thank You!

◆ **References**


